Paper-II: Recent Advances in Bio-Mathematical Modelling (Sumaira Anjuma & Sabira Ali)

Max. Marks: 100 **Time: 3 hours**

Unit I:
Linear and non-linear growth and decay models. Thermoregulation in human body, models of heat transfer, heat governing equation- bioheat equation. Mass balance equation, equation of continuity in fluids, movement balance equation, Euler-Lagrange equation and variational integrals, finite element method, finite volume method, solution of heat and mass equations using FEM.

Unit II: The role of transport processes in biological systems: Diffusion, convection, transport within the cell, across cell membrane. Physiological transport Systems: Caridovascular system and transport through kidneys. Diffusion of glucose or medicine in the blood stream. The circulatory system, blood flow, cardiac output, Stewart-Hamilton method for measuring cardic output.

Unit III: Conservation relations and momentum balances: Fluid kinematics, conservation relations and boundary conditions, Newton's law of viscosity, Measurement of blood viscosity, regulation of blood flow, Poiseulle's law and blood transport models. Conservation relations for fluid transport: General form of equation for conservation of mass, and linear momentum.

Unit IV: Fluid flow in the circulation and tissues: Oscillating flow in a cylindrical tube, flow in curved vessels, flow in branching vessels, flow in specific artries. Transport in porous media: Fluid flow in porous media, Darcy's law, Brinkman equation, Squeeze flow, solute transport in porous media.

Refereneces

1. Introduction to Finite Element Method, J N Reddy, Oscar S. Wyatt Chair in Mechanical Engineering. Texas A & M University.
2. Transport phenomena in Biological systems, George A. Truskey, Fan Yuan, David F. Katz, Pearson Prentice Hall, New Jersy.
3. Mathematical Models of Biological Systems; Hugo van den Berg; Oxford University Press.
4. Introduction to Mathematical Modelling and Biomathematics; M. A. Khanday; Dilpreet Publishing House , New Delhi.
5. Mathematical Models in Biology; Elizabeth S Allman, John A Rhodes.
6. A Textbook of Medical Physiology 8th edition; Guyton; W B Saunders Pub.
7. Emerging Topics In Heat and Mass Transfer in Porous Media; Peter Vadasz; Springer.
8. Essiantials of Human Physiology ; Noel Paton ; W T Keener and Co, Chicago.
9. Biological and Bioenvironmental Heat and Mass Transfer; Ashim K Datta; New York, Basel.
10. Mathematical Physiology Vol. I-III ; James Keener, James Sneyd; Springer.

Paper-III : Research Topics in Mathematical Physiology (Sabira Ali)

Max. Marks: 100 **Time: 3 hours**

Unit I: Mass transport and biochemical interactions: Chemical kinetics and reaction mechanisms, reaction rates, first order reactions, second order irreversible reactions, reversible reactions, enzyme kinetics, derivation of Michelis-Menton kinetics, derivation of K_M and R_{max}, Competitive inhibition, uncompetitive and non-competitive inhibition, substrate inhibition, effect of diffusion and convection on chemical reactions.

Unit II: Blood plasma, blood cell production, periodic hematoligical disease, a simple model for blood cell growth, erythrocytes, myoglobin and haemoglobin, saturation shifts, O_2 and CO_2 transport.

1. K. N. Chao, J. C. Eisely and W. J. Yang (1973), *Heat and Water Migration in Regional Skin and Subcutaneous tissues,* Bio-Medical Sym. ASME, pp. 69-72.

Unit III: Drug transport in solid tumors, drug delivery in cancer treatment, routes of drug administration, quantative analysis of transvascular transport, interstitial fluid transport, governing equation, intertitial hypertension in solid tumors. Simple compartment model in pharmacokinetic analysis, one compartment and two compartment models, transport of medcine in individual organs. Transdermal drug delivery system.

Unit IV:

1. Pontrelli G, Mascio AD, de Monte F. (2013). Modelling transdermal drug delivery through a two-layered system. Simultech, pp. 645–651.
2. Khanday MA, Rafiq A. (2016). Numerical estimation of drug diffusion at dermal regions of human body in transdermal drug delivery system. J Mech Med Biol. 16(03):1650022.
3. Khanday MA, Rafiq A, Nazir K. (2017). Mathematical models for drug diffusion through the compartments of blood and tissue medium. AJM Elsevier. 53(3): 245–249.
4. Saqib Mubarak & M. A. Khanday (2021): Mathematical modelling of drug-diffusion from multi-layered capsules/tablets and other drug delivery devices, Computer Methods in Biomechanics and Biomedical Engineering, DOI: 10.1080/10255842.2021.1985477

References:

1. Mathematical Models of Biological Systems; Hugo van den Berg; Oxford University Press.
2. Introduction to Mathematical Modelling and Biomathematics; M. A. Khanday; Dilpreet Publishing House , New Delhi.
3. Mathematical Models in Biology; Elizabeth S Allman, John A Rhodes.
4. A Textbook of Medical Physiology 8th edition; Guyton; W B Saunders Pub.
5. Emerging Topics In Heat and Mass Transfer in Porous Media; Peter Vadasz; Springer.
6. Essiantials of Human Physiology ; Noel Paton ; W T Keener and Co, Chicago.
7. Biological and Bioenvironmental Heat and Mass Transfer; Ashim K Datta; New York, Basel.
8. Mathematical Physiology Vol. I-III ; James Keener, James Sneyd; Springer.

Paper-III : Research Topics in Mathematical Physiology (Sumaira Anjum)

Max. Marks: 100 **Time: 3 hours**

Unit I: Mass transport and biochemical interactions: Chemical kinetics and reaction mechanisms, reaction rates, first order reactions, second order irreversible reactions, reversible reactions, enzyme kinetics, derivation of Michelis-Menton kinetics, derivation of K_M and R_{max}, Competitive inhibition, uncompetitive and non-competitive inhibition, substrate inhibition, effect of diffusion and convection on chemical reactions.

Unit II: Allosteric enzymes, the symmetry model, the sequential model and negative cooperativity, physiological aspects of cooperativity, analysis of cooperativity. Kinetic treatment of allosteric enzymes, kinetic cooperativity and Slow transition model, pH and temperature dependenc of enzymes, stability and determination of pK values, Statistical methods in enzyme kinetics. Cooperativity of enzymes and saturation function, Dimer, trimer and tetramer, hemoglobin and its different states.

Unit III

1. Illeova, V., Polakovic, M., Vladimir, S., Acai, P. and Mohammad, J. (2003). Experimental Modeling of thermal inactivation of urease, Journal of Biotechnology, Vol. 105, pp. 235-243.
2. Ananthi, S.P., Manimozhi, P., Praveen, T., Eswari, A. and Rajendran, L. (2013). Mathematical modeling and the analysis of the kinetics of thermal inactivation of enzyme, Hindawi Publishing Corporation, pp. 1-8.
3. Eskandari, E.M. and Taghizadeh, N. (2020). Exact solutions of two nonlinear space-time fractional differential equations by applications of Exp-Function method, Applications and Applied Mathematics, Vol. 15, pp. 970-977.
4. Khanday, Mukhtar Ahmad and Bhat, Roohi (2021). (R1488) Transformation of Glucokinase under Variable Rate Constants and Thermal Conditions: A Mathematical Model, Applications and Applied Mathematics: An International Journal (AAM), Vol. 16 (2), https://digitalcommons.pvamu.edu/aam/vol16/iss2/10

Unit IV: Multiple Equilibria: Diffusion, interaction of ligands and macromolecules, derivation of binding equation, macromolecules with identical independent binding sites, non-identical independent binding sites. Macromolecules with identical, interacting binding sites: The Hill equation, the Adair equation, the Pauling model.

Refereneces

1. Enzyme Kinetics, Principles and Methods, Hans Bisswanger,Wiley-VCH Verlag Germany.
2. Mathematical Models of Biological Systems; Hugo van den Berg; Oxford University Press.
3. Introduction to Mathematical Modelling and Biomathematics; M. A. Khanday; Dilpreet Publishing House , New Delhi.
4. Mathematical Models in Biology; Elizabeth S Allman, John A Rhodes.
5. A Textbook of Medical Physiology 8th edition; Guyton; W B Saunders Pub.
6. Essiantials of Human Physiology ; Noel Paton ; W T Keener and Co, Chicago.
7. Mathematical Physiology Vol. I-III ; James Keener, James Sneyd; Springer.

Sajid Mushtaq
Zahoor Iqbal Bhat

Paper II: Advanced Topics in Matrices and Graphs

Maximum Marks: 100 Time 3 hours

There will be four questions, one question from each unit with internal choice and the student will be asked to attempt all the four questions. Each question will carry 25 marks.

Unit 1: Unitary equivalence, Schur's unitary triangularization and its implications, Spectral theorems for normal and Hermitian matrices. Gershgorin discs with associated perturbation theorems and inclusion results. Jordan canonical forms with application, minimal polynomials and companion matrices.

Unit 2: Variational characterizations of eigenvalues of Hermitian matrices, Rayleigh-Ritz theorem, Courant-Fischer theorem, Weyl theorem, Cauchy interlacing theorem, Inertia and congruence, Sylvester's law of inertia. Matrix norms, spectral radius formula and relationships between matrix norms.

Unit 3: Positive definite matrices, characterizations of definiteness, polar form and singular value decompositions, congruence and simultaneous diagonalization. Non-negative matrices, Positive matrices, Perron-Frobnus theory, Irreducible non-negative matrices and primitive matrices.

Unit 4: Adjacency matrix of a graph, Powers of Adjacency matrix of a graph, Sach's Theorem for computing coefficients of characteristic polynomial of a graph, Determination of spectra of some special graphs like K_n, C_n, P_n. Bipartite graphs and their spectral characterization. Spectrum of regular graphs, strongly regular graphs and their characterization, Characterization of connected regular graphs, Relation between number of distinct eigenvalues and diameter of a graph. Spectrum of complements of regular graphs, Spectrum of line graphs of regular graphs.

References

1. Horn and Johnson, Matrix Analysis, Cambridge: Cambridge University Press (1990).
2. Topics on Matrix Analysis, Horn and Johnson (1991).
3. Brouwer and Haemers, Spectra of Graphs, Springer, New York (2012).
4. R. B. Bapat, Graphs and Matrices, London: Springer (2011).
5. X. Li, Y. Shi, I. Gutman, Graph energy, Springer-Verlag, New York (2012).
6. D. B. West, Introduction to Graph Theory, Prentice Hall (2001).
7. N. Giggs, Algebraic Graph Theory, Cambridge: Cambridge University Press, London (1993).

Paper III: Research topics on spectral graph theory

Maximum Marks: 100 Time 3 hours

There will be four questions with internal choice and the student will be asked to all four questions. Each question will carry 25 marks.

Unit 1: Eigenvalues of the Laplacian of a graph (Anderson-Morley), on the Laplacian eigenvalues of a graph (Li-Zhang), a note on Laplacian eigenvalues of a graph (Merris), The second largest Laplacian eigenvalues of graphs, on Laplacian eigenvalues of a graph (Zhao), a characterization on graphs which achieve upper bound (Das). Laplacian spectrum of a graph (Das), a sharp upper bound on the largest eigenvalue of Laplacian matrix of graph (Shu et.al), Laplacian energy (Gutman-Zhao), bounds for Laplacian energy (Gutman-Zhao, Das-Mojallal)

- W.N. Anderson and T.D. Morley, Eigenvalues of the Laplacian of a graph, Linear and Multilinear algebra 18(1985) 141-145.
- J.S. Li and X.D. Zhang, On the Laplacian eigenvalues of a graph, Linear Algebra Applications, 285 (1998) 305-307.
- R. Merris, A note on Laplacian graph eigenvalues, Linear Algebra applications 285 (1998) 33-35.
- L.S. Li and Y.L. Pan, A note on the second largest eigenvalue of the Laplacian matrix of a graph, Linear Multilinear Algebra, 48 (2000) 117-121.
- I.M. Guo, On the second largest Laplacian eigenvalue of trees, Linear Algebra Applications 404 (2005) 251- 261.
- J. L. Shu, Y. Hong and K.W. Ren, A sharp upper bound on the largest eigenvalue of Laplacian matrix of graph, Linear algebra application 347 (2002) 123-129
- B. Zhou, On Laplacian eigenvalues of graph, Z. Naturforsch 59a (2004) 181-184.
- K.C. Das, A characterization on graphs which achieve upper bound for the largest Laplacian eigenvalue of graph, Linear algebra application, 376(2004)173-186.
- K.C. Das, the Laplacian spectrum of graph, computer and Mathematics with application, 48(2004)715-724.

Unit 2: Graphs with $n-1$ main eigenvalues (Du et. al) and adjacency eigenvalues of graphs without short odd cycles (Li et. al). Spectral radius and matchings in graphs (O. Suil), spectral radius and degree deviation in graphs (Zhang), Spectral radius and clique partitions of graphs (Zhou et. al) and spectral radius of graphs without star forest (Chen et. al). Proof and disproof of conjectures on spectral radii of coclique extension of cycles and paths (Sun et.al).

- M Chen, A. Liu and X. Zhang, On the spectral radius of graphs without star forest, discrete Math. 344 (2021)112269.
- Z. Du, F. Liu, S. Liu and Z. Qin, Graphs with n-1 main eigenvalues.
- S. Li, W. Sun, and Y. Yu, Adjacency eigenvalues of graphs without short odd cycles, arXiv, 2021.
- O. Suil, Spectral radius and matchings in graphs, Linear Algebra Applications 614 (2021) 316-324.

Paper III: Research topics on spectral graph theory

Unit 1: Laplacian matrix, Signless Laplacian matrix of a graph, Basic properties, Computing Laplacian eigenvalues, Cauchy Binet theorem, Matrix tree theorem and computing number of spanning trees of $K_n, K_{m,n}$, complete multipartite graph, $L(K_n)$, d −dimensional cube, Algebraic connectivity, preliminary results, Classification of trees, Monotonicity properties of Fiedler vector

Unit 2: Signed graphs, Basic definitions, Balance, Harary's Balance theorem, Anti-balance and its characterization, switching equivalence, switching isomorphism, Adjacency matrix, powers of adjacency matrix, matrix version of switching, properties of switching, Acharya's spectral criterion for balance for weighted directed graphs. Orientation in signed graphs, bi-directed graphs. Incidence matrix, Laplacian matrix of signed graphs and its basic properties.

Unit 3: Spectral properties of signed and weighted digraphs, Analogue of Sach's theorem for signed/weighted digraphs, Alternating property of coefficients in bipartite signed digraphs/weighted digraphs, Signed digraphs/weighted digraphs with three distinct eigenvalues.

Unit 4: Energy of graphs, Coulson integral formula for energy, McClelland Inequality for the energy of graphs, Energy of digraphs and signed digraphs, Unicyclic and bicyclic digraphs/signed digraphs with extremal energy, McClelland inequality for energy of digraphs/signed digraphs. Energy in weighted digraphs, Upper and lower bounds on the energy of weighted digraphs.

References

1. F. Belardo and S. K. Simić, On the Laplacian coefficients of signed graphs, Linear Algebra and its Applications, 475 (2015) 94-113.
2. I. Pena and J. Rada, Energy of digraphs, Linear and Multilinear Algebra, 56(5) (2008) 565-579.
3. J. Monsalve and J. Rada, Bicyclic digraphs with maximal energy, Applied Mathematics and Computation, 280 (2016) 124-131.
4. M. A. Bhat, Energy of weighted digraphs, Discrete Applied Mathematics, 223 (2017) 1-14.
5. M. A. Bhat and J. Rada, Lower bounds for the energy of weighted digraphs, 67(4) (2019) 743-755.
6. M. A. Bhat and S. Pirzada, Spectra and energy of bipartite signed digraphs, Linear and Multilinear Algebra, 64(9) (2016) 1863-1877.
7. N. Giggs, Algebraic Graph Theory, Cambridge: Cambridge University Press, London (1993).
8. R. B. Bapat, Graphs and Matrices, London: Springer (2011).
9. S. Pirzada and M. A. Bhat, Energy of signed digraphs, Discrete Applied Mathematics, 169 (2104) 195- 205.
10. T. Zaslavsky, Matrices in the theory of signed simple graphs, Advances in Discrete Mathematics and Applications: Mysore, Ramanujan Math. Soc., Mysore, (2010) 207-229.

- S. Sun and K. C. Das, Proof and disproof of conjectures on spectral radii of coclique extension of cycles and paths, Linear Algebra Applications 618 (2021) 1-11.
- J. Zhou and E. Dam, Spectral radius and clique partitions of graphs, Linear Algebra Applications 630 (2021) 84 -94.
- W. Zhang, A note on spectral radius and degree deviation in graphs, Discrete Math. 344 (2021) 112429.

Unit 3: On the sum of Laplacian eigenvalues of graph (Hammers et. al), on the sum of Laplacian eigenvalues of tree (Fritscher. et. al), on a conjecture for the sum of Laplacian eigenvalues (Wang et. al), an interlacing approach for bounding the sum (Abail et. al), on the distribution of Laplacian eigenvalues of tree (Braga et. al).

- A. Abiad, M. A. Fiol, W.H. Haemers and G. Perarnau, An interlacing approach for bounding the sum of Laplacian eigenvalues of graphs.
- R.O.Braga, V.M.Rodrigues and V.Trevisan, On the distribution of Laplacian eigenvalues of trees, Discrete Math, 313 (2103) 2383-2389.
- Z. Du and Bozhou, Upper bounds for the sum of Laplacian eigenvalues of graphs, Lines Algebra Applications, 436 (2012) 3672-3683.
- E. Fritscher, C. Hoppen, I. Rocha and V. Trevisan, On the sum of Laplacian eigenvalues of a tree, Linear Algebra Applications, 435(2011) 371-399.
- W. H Heamers, A. Mohamadian and B. Tayfie-Rezaie, On the sum of Laplacian eigenvalues of graph, linear algebra application, 432(2010), 2214-2221.
- Ji.MingGuo, Xiao . Li. Wu and Jing. Ming. Zang, On the distribution of Laplacian eigenvalues of graph, Acta. Math. Sinica. English series. 27(11)(2011), 2259-2268.
- S. Wang, Y. Huang and B. Liu, On a conjecture for the sum of Laplacian eigenvalues values, Math computer Modelling, 56(2012) 60-68.

Unit 4: The maximum spectral radius of C_4-free graphs of given order and size (Nikiforov), The spectral Turán problem about graphs with no 6-cycle (Zhai et. al.), A Spectral Condition for the Existence of the Square of a Path (Zhao et. al.), Note on the sum of the smallest and largest eigenvalues of a triangle-free graph (Péter Csikvári), The maximum spectral radius of graphs with forbidden subgraph (Fang et. al.).

- P. Csikvári, Note on the sum of the smallest and largest eigenvalues of a triangle-free graph, Linear Algebra and its Applications, (2022) 650, 92-97.
- X. Fang, L.You, The maximum spectral radius of graphs of given size with forbidden subgraph, Linear Algebra and its Applications, (2023) 666, 114-128.
- V. Nikiforov, The maximum spectral radius of C_4-free graphs of given order and size. Linear algebra and its applications, 430(11-12), 2898-2905.
- M. Zhai, B. Wang and L. Fang, The spectral Turán problem about graphs with no 6-cycle. Linear Algebra and its Applications, (2020) 590, 22-31.
- Y. Zhao, J. Park, A Spectral Condition for the Existence of the Square of a Path, Graphs and Combinatorics, (2022) 38(4), 126.