CONTRIBUTION OF SOME EMINENT FEMALE MATHEMATICIANS

Project Work

Submitted

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STUDENT DECLARATION

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We further declare that the work reported in this project has not been submitted and will not be submitted for any other degree of this University or any other University.

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CERTIFICATE OF ORIGINALITY

This is to certify that the project dissertation entitled "**Contribution Of Some Eminent Female Mathematicians"** submitted by Aatira Hilal, Athar-un-Nisa, Khan-Qurat-ul-Aein, Saima Bashir, Uzma Muzafar, Asmat Habib, Nahida Bano and Yasmeena Manzoor in partial fulfillment of the requirement for the award of **Master's Degree In Mathematics,** fulfills all the prescribed norms given in the statutes and ordinances of **University of Kashmir**

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Chapter 1

1.1 Introduction

It becomes quite pertinent to understand the various meanings of the term "Mathematics" which is a part of our general education.

The term 'Mathematics' is defined in different ways by different writers. To begin with, the word 'Mathematics' comes from a Greek word, meaning 'inclined to learn'. The Oxford Guide to the English Language (1984) defines mathematics as "the science of numbers, quantities and measurements."To Locke, mathematics was

"A way to settle in mind the habit of reasoning".

Lindsay believed that "Mathematics is the language of physical sciences and certainly no more marvelous language was ever created by the mind of man." So we can say that Mathematics is the branch of science which deals with the study of numbers, notations, quantities, measurements etc. and how they affect each other when we subject all these definitions to analysis.

Mathematics is a very broad term and it touches the various aspects of human knowledge and learning. Fundamentally, Mathematics is a science related to space and numbers, to quantify facts and relationships. It opens the gates of nature to allow us to derive maximum benefits for mankind. Whether the program of modern mathematics is successful or not is not relevant. But quantitatively better mathematical education is undoubtedly the need of the day. Mathematics enriches our imagination and modes of thinking and even behavior to excel in life.

Modern civilization is the gift of Mathematics. For scientists, technologists, engineers, doctors, specialists and others, Mathematics is there to have catalytic impact upon their understanding in order to enrich their performance to serve mankind all over the globe, productively and usefully. This is the work of our mathematicians to improve the quality of human life by reducing time, distance and human effort. For instance, computer, the new magic of mathematics, has a wonderful contribution towards human growth and development. Weather forecasts, diagnosing diseases, finding hidden wealth under the soil, injecting fool proof precision in our industrial production, revolutionizing agriculture etc. are modern gifts to people everywhere, simplified by the mathematical understanding and application. In fact Mathematics has been in the veins of great thinkers. It is fair to recall the great German mathematician, David Hilbert, who trumpeted his observations by saying

"*Mathematics is what competent people understand the world to mean*"

Modern mathematics enables boys and girls to cope up with the challenges of the 'world of work' where situations cannot be handled in a comfortable manner. It is rightly said that

Every part of our lives involves mathematics".

The knowledge and skill of Mathematics is employed to understand natural phenomena like sunrise and sunset, change of seasons, eclipses, rotation of planets, earthquakes, volcanic explosions, space, deep sea investigations and so on, to make human life richer and safer. Advances in science and technology come as a blessing, emanating from the application of Mathematics to usher in a better quality of life for the people of the world. It follows that the Mathematics is without an iota of doubt, a very promising subject in view of its practical utility in all walks of life. So impressed was Napoleon Bonaparte that he said.

"The progress and the improvement of mathematics are

linked to the prosperity of the state".

Hamilton says,

"*the study of mathematics cures the vice of mental distraction and*

cultivates the habit of continuous attention."

"Mathematics possesses art, beauty, music and finesse for its lovers." It is the sacred duty of the mathematics teachers to take their students as lovers of mathematics, nearer to these so as to have four-fold benefits for enjoying its art, beauty, music and above all its finesse. It is well said,

"*Mathematics is science of all sciences and art of all arts"*

Great mathematicians and researchers have followed the path of intellectual discipline to put forth new discoveries and inventions at the disposal of society. It is said,

"*nature never did betray the heart that loved her*"

and the same nature has been made friendly to us by discoveries and inventions of many mathematicians known to us. James watt's steam engine, Galileo's clock and telescope, Newton's Law of gravity, etc. are the gifts of mathematics pressed into service for the cultural growth of human civilization by the great mathematical personalities. So it sounds very well when Hugben says

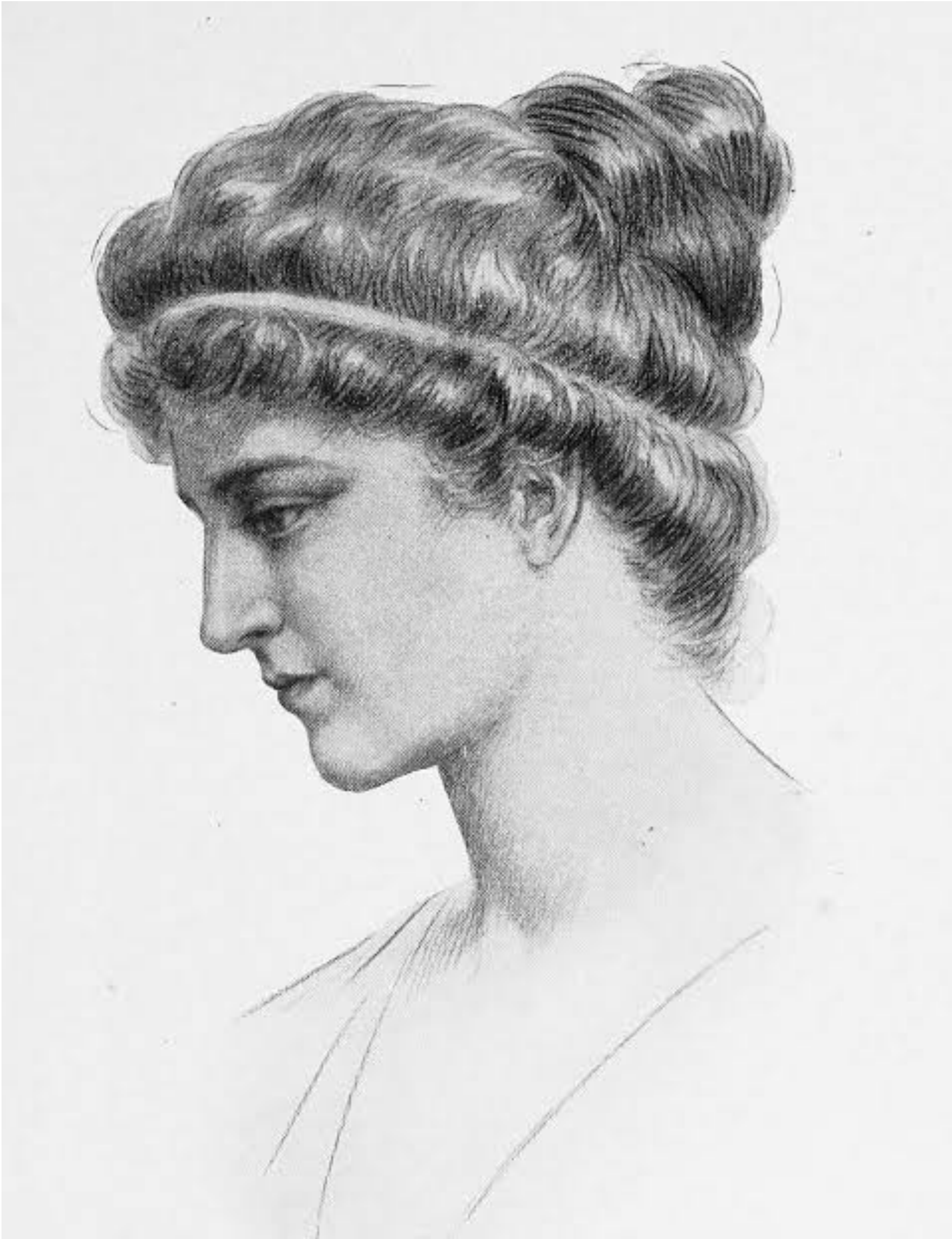
"*Mathematics is the mirror of civilization*"

Mathematics originated in a very few areas, within the frame work of highly developed urban civilizations and well organized economic conditions. The very idea of history leads our attention to the times immemorial when ancient shepherds who lived as wanderers without any town or village, used to perform simple calculations and counting with the aid of figures and stones. By noticing the length of shadows, they could roughly tell the time of the day. This clearly shows how profoundly the people of remote past were influenced by mathematics although it was hidden and indirect. They had their limitations, no doubt, most of them made astonishing and interesting mathematical discoveries.

It is no exaggeration to say that history of mathematics is the history of civilization. Our entire present civilization, as far as it depends upon the intellectual penetration and utilization of nature, has its real foundations in the mathematical sciences. But it is disheartening that the present curriculum in Mathematics does not care for the introduction of the study of the history of Mathematics simply because no time can be found for its study. Once introduced, it is sure to become a source of interest and pleasure to the learners.

Thus it is clear that mathematicians throughout the history of mankind have put in their blood and sweat for the betterment of mankind.

To immortalize the contributions made by the mathematicians towards the subject, several awards are distributed among distinguished mathematicians internationally. Interestingly, there is no noble prize awarded to mathematicians but an equally prestigious award called as **Fields Medal** named after Canadian mathematician John Charles Fields is awarded to two, three or four mathematicians under 40 years of age, after every four years. It is presented by International Mathematical Union (IMU). Another award known as **Abel prize** named after Norwegian mathematician Niels Henrik Abel is awarded annually by the King of Norway to distinguished mathematicians along with a prize of 7.5 million Norwegian kroner. Breakthrough prize in Mathematics, Paul Erdős award, David Hilbert Award, Churn Medal are some among various other prizes awarded to mathematicians to appreciate their contribution. The better someone was with numerical calculations, the better they were at regulating fear and anger. Strong math skills may even be able to help treat anxiety and depression.



Hypatia (born c.350-370; died 415AD)

Chapter 2

Hypatia of Alexandria

2.1 Life History

Hypatia of Alexandria was a renowned philosopher, mathematician and astronomer who lived in Alexandria, Egypt during the late fourth and early 5th century. She is considered one of the leading intellectuals of her time.

Regarding Hypatia's birth date, a lack of conclusive historical events has left the data of Hypatia's birthday open to conjuncture. The most commonly cited date is 370 CE, given by the German Classicist Richard Hoche in 1860 based on the interpretation of Hypatia's biography in the Suda lexicon, a 10th century Byzantine encyclopedia. Since the 1980's this date has been revisited by scholars who argue that Hoch's interpretation of the Suda Lexicon is based on several incorrect assumptions and that an earlier date is more acceptable. In support of an earlier birth date of Hypatia, it has been noted that Synesius of Cyrene, Hypatia's most prominent student, is believed to have been born about 370 CE, and he must have been significantly younger than his teacher. Also, the 6th century chronicler John Malalas wrote that Hypatia was an old woman at the time of her death, 415 CE.

Michael Deakin and Maria Dzielska, two later biographers of Hypatia, both accept 355 CE as an approximate birth date". Now it is believed that Hypatia was born around 355 CE in Alexandria which was the prominent centre of learning and intellectual exchange in the ancient world. Her father *Theon of Alexandria* was a mathematician and philosopher who played a crucial role in her education. Under his guidance Hypatia received a comprehensive education that included mathematics, astronomy, philosophy and literature. Theon is best remembered for the part he played in the preservation of 'Euclid's Elements' but he also wrote extensively, commenting on Ptolemy's Almagest and Handy Tables. Hypatia continued his program, which was essential and determined effort to preserve the Greek mathematical and astronomical heritage in extremely difficult times. She is credited with commentaries on Apollonius of Pergo's Conics (geometry) and Diophantus of Alexandria's An Astronomical Table (possibly revised version of book III of her father's commentary of the Almagest) these works, the only ones she is listed as having written, have been lost although there have been attempts to reconstruct aspects of producing her commentaries on Apollonius and Diophantus, she was pushing the program initiated by her father in to more recent and more difficult areas.

The Christian historian, Socrates of Constantinople, a contemporary of Hypatia, describes her in his Ecclesiastical History "There was a woman at Alexandria named Hypatia, daughter of the philosopher Theon, who made such attainments in literature and science, as to far surpass all the philosophers of her own time. Having succeeded in the school of Plato and Plotinus, she explained the principles of philosophy to her auditors, many of whom came from a distance to receive her instructions. On account of the self-possession and ease of manner which she had acquired in consequence of the cultivation of her mind, she not infrequently appeared in public in the presence of the magistrates. Neither did she feel abashed in going to an assembly of men. For all men on account of her extraordinary dignity and virtue admired her more."

Hypatia's passion for knowledge and her exceptional intellectual abilities quickly became apparent. She immersed herself into various branches of learning, mastering the works of ancient philosophers like Plato and Aristotle and making significant contribution to mathematics and philosophy. Although much of her work has been lost, her inference and legacy continues to inspire scholars and thinkers of this day. As women in a male dominant field, Hypatia faced challenges and prejudices. However, her reputation as brilliant scholar and her dedication towards her studies allowed her to gain respect and recognition among her peers. She even became the head of the Neoplatonic School in Alexandria a position previously held by her father. Hypatia's influence extended beyond the academic realm, she was known for her eloquence, wit and moral integrity. She advocated for intellectual freedom and critical thinking, emphasizing the importance of questioning and seeking truth through reason. Her teachings and philosophical ideas resonated with many, but they also attracted enemies and detractors.

In 415 CE tension arose in Alexandria between Christians and non Christian intellectuals and her perceived influence over political matters made her target for religious zealots. A mob led by a fanatical Christian sect attacked her, accusing her of witch craft and impiety. Hypatia was brutally murdered and her body was mutilated and burned. Hypatia's tragic death marked the end of an era of intellectual freedom and tolerance in Alexandria. Her murder became a symbol of a conflict between religious fanaticism and the pursuit of knowledge. Despite her untimely demise, Hypatia's ideas and contributions have continued to inspire generations of scholars and thinkers. She remains an icon of intellectual courage and the pursuit of truth in the face of adversity.

As the head of the Neoplatonic School, Hypatia played a crucial role in educating and inspiring her students. Her lectures and teachings attracted scholars from various backgrounds and she was known for her engaging style and ability to communicate complex ideas clearly. Hypatia's mentorship and guidance shaped the intellectual development of many aspiring mathematicians, astronomers and philosophers.

2.2 Contributions

Hypatia of Alexandria made significant contributions to the field of mathematics, astronomy and philosophy. Although much of her work has been lost, her influence and legacy can be seen in various ways.

Hypatia was a renowned mathematician who made important advancements in the field. She expanded upon the work of previous mathematicians such as Euclid and Diophantus, Hypatia's contributions to mathematics include advancement in algebra, geometry and number theory. She developed new methods and techniques for solving mathematical problems and wrote commentaries on existing mathematical works, providing valuable insights and interpretations.

Hypatia was also a skilled astronomer. She studied celestial phenomena, observed the movements of planets and made calculations related to astronomical events. Hypatia's contributions in astronomy were significant and she likely developed her own theories and models to explain the motions of celestial bodies. Unfortunately, most of her astronomical writings have been lost and we have limited knowledge of her specific contributions in this field.

Hypatia was associated with the Neoplatonic School of philosophy, which was a blend of Platonic and Aristotelian ideas. She expanded on the earlier philosophers and developed her own philosophical ideas. Hypatia emphasized the importance of reason, critical thinking and intellectual freedom. She encouraged her students to question and explore ideas seeking truth through logical inquiry. Her philosophical teachings had a profound influence on her students and the wider intellectual community.

Some of Hypatia's work which had been lost is mentioned below.

A commentary on Archimedes's Sphere and Cylinder surviving as John of Tynemouth's De Curvis Superficibus, a text on isoperimetric figures incorporated by a later author into Introduction to the Almagest.

A commentary on Archimedes's Dimension of the Circle.

A commentary edition of Apollonius Pegasus's Conics upon which later commentary editions were based.

She also created an "Astronomical Canon", which is believed to have been either a new edition of the Handy Tables by Alexandrian Ptolemy or a commentary on his book Almagest.

Maria Gaetana Agnesi (1718-1799)

Chapter 3

Maria Gaetana Agnesi

3.1 Life History

Maria Gaetana Agnesi was born in Milan Italy on 16^{th} May 1718 in a wealthy family. Maria Gaetana Agnesi was an Italian mathematician and philosopher who made noteworthy contributions to Differential equation, Calculus and Algebra. Maria Gaetana Agnesi is noted as the first woman to be appointed as a chair of mathematics at a university and was the first woman to write a math textbook “Basic Principle of Analysis”, this text book was one of the first written in the field of calculus and was translated and adopted by scholars across Europe. In addition to these historic milestones, Agnesi lived a life dedicated to helping women and the poor. She opened the hospital for the poor in her home, gave away most of her wealth and advocated for educational equality for women, arguing that women should be free to pursue any knowledge that was available to men.

Her father Pietro Agnesi was an ambitious man and wanted to raise his family to the rank of Milanese nobility. Maria was the eldest child of Pietro Agnesi and Anna Fortunato Brivio, Maria Agnesi subsequently became one of the 21 children by three wives of her father. The Agnesi family made their money from silk and not from Pietro’s alleged position as a professor of mathematics which did not exist. Her father’s quest for social status shaped much of Maria’s childhood. Her parents provided her with good early education. When her mother died in 1732, Maria, being the eldest child from a large family, retired from public life and stayed to manage the house. She was not only a math genius but she also proved to be a very kind and religious woman who did her part in helping people and keeping her faith and she was also a linguist, speaking several languages fluently. Maria Gaetana Agnesi showed signs of extraordinary intelligence early on in life and she was recognized as a child prodigy. She knew how to speak Italian and French at the age of six. At the age of 11 she was fluent not only in Italian and French but she could also speak Latin, German, Greek, Hebrew and Spanish. Maria was also called the “Seven Tongued Orator”. When she was 9 years old she amazed some distinguished minds of their day by composing a speech in Latin which lasted an hour long. She talked about the rights of women to obtain education. At the age of 12 Maria was struck by an illness no one could identify. However, doctors pointed her excessive studying and reading as a cause and so she was advised to be more active and go to horseback rides and to dance. Dancing and horseback riding did not work and she still suffered from convulsions so she was told to practice everything in moderation.

She was a brilliant child who did her part to help educate her younger brothers. At the age of 14, she was studying geometry and ballistics. She was well educated and received recognition for her intellectual accomplishments, being the first woman appointed as a mathematics professor at the university of Bologana in 1750. When she was 15 years old her father began to regularly gather in his house a circle of the most learned in Bologana, before whom she maintained a series of theses on the most abstruse philosophical questions. Records of these meetings are given in *Charles de Brosse's Letters sur l'ltalie* and in the *Propositions Philosophicae,* which her father had published in 1738 as an account of her final performance, where she defended 190 philosophical theses. Her father agreed with her that if she were to continue her research into mathematics, then she would be permitted to do all the charity work she wanted. It is worth noting that while she was brilliant, Agnesi was shy and did not really relish being put in display or asked to talk in front of a group. Maria wanted to join a convent but her father did not allow it. After his death in 1752, she turned all of her efforts to charity work and lived like a nun until her own death in 1799.

3.2 Contributions

Maria Gaetana Agnesi made a significant contribution in the field of mathematics, particularly in the 18th century. Her most notable work is in the field of calculus and mathematical analysis. Here are some of her key contributions:

Agnesi Curve (Witch of Agnesi)

She investigated and popularized the curve known as "witch of Agnesi" or "Versiera" (or simply the witch), in 1748 in her book *Instituzioni Analitiche Ad Uso Della Gioventu Italiana* (the first surviving mathematical work written by a woman). The curve is also known as *cubique d'Agnesi* or *Agnesienne*, which is a cubic curve defined by an equation. This curve was studied extensively and named in her honor. This curve was originally described and had been studied earlier by Pierre de Fermat and Guido Grande in 1703. The curve has applications in geometry and calculus.

Contribution to Differential Equations

Agnesi worked on differential equations, which are fundamental in mathematics and science. Her research in this area contributed to the understanding of these equations and their applications.

Contribution to Calculus

Agnesi's work touched the study of curves and their properties. She examined the behavior of functions and their derivatives, which was instrumental in the development of calculus.

Recognition and Influence

Agnesi's contributions earned her recognition and respect in the academic and mathematical communities of her time. She corresponded with other prominent mathematicians of the era, such as Euler and her work influenced the development of calculus and mathematical education.

Instituzioni Analitiche

She authored a comprehensive mathematics textbook titled "Analyticae ad curvilineas descendents et Ascendants" (Analytical Institutions for the Descending and Ascending Lines). This pioneering work covered a wide range of mathematical topics, including algebra, calculus and differential equations. It played a role in the development and dissemination of calculus.

Mathematical Conversations

Maria held weekly gathering, known as "Academy of the Pious", where she discussed mathematical and philosophical topics with other scholars.

Mathematical Treatises

She wrote various mathematical treatises and essays on subjects like differential calculus, algebra and the curve of transcendental equations.

Promoting Education

Agnesi was an advocate for education, particularly the education of women. Her dedication to teaching and writing text books which aimed to make mathematics more accessible was influential in her time.

Academic Recognition

Agnesi was a first woman to be appointed as a mathematics professor at the University of Bologna in 1750, breaking gender barriers in academia.

While her work on the "witch of Agnesi" and her textbook are the most renowned, Maria Gaetana Agnesi's overall contributions to mathematics and her role in promoting the study of mathematics for both genders were significant and influential during her time and beyond. Although her mathematical career was relatively short, Maria Gaetana Agnesi's work significantly impacted the field of mathematics, especially for breaking the gender barriers in academia and she remains an inspirational figure for women in STEM fields.

Quotes by Maria Agnesi:

1. "I hope my studies have brought glory to God, as these were useful to others and derived from obedience, because that was my father's will. Now I have found better ways and means to serve God, and to be useful to others.
2. "Analytics is the art of resolving all kind of mathematical questions, by finding or computing unknown numbers, or quantities, by the means of others that are known or given".

3.3 Prizes and Awards:

Maria received a gold medal and a gold wreath adorned with precious stones presented by Pope Benedict XIV in honor of her publication of Instituzioni Analitiche (1749); Crystal box with diamonds and a diamond ring by Empress Maria Theresa of Austria to whom Instituzioni Analitiche was dedicated (1749).

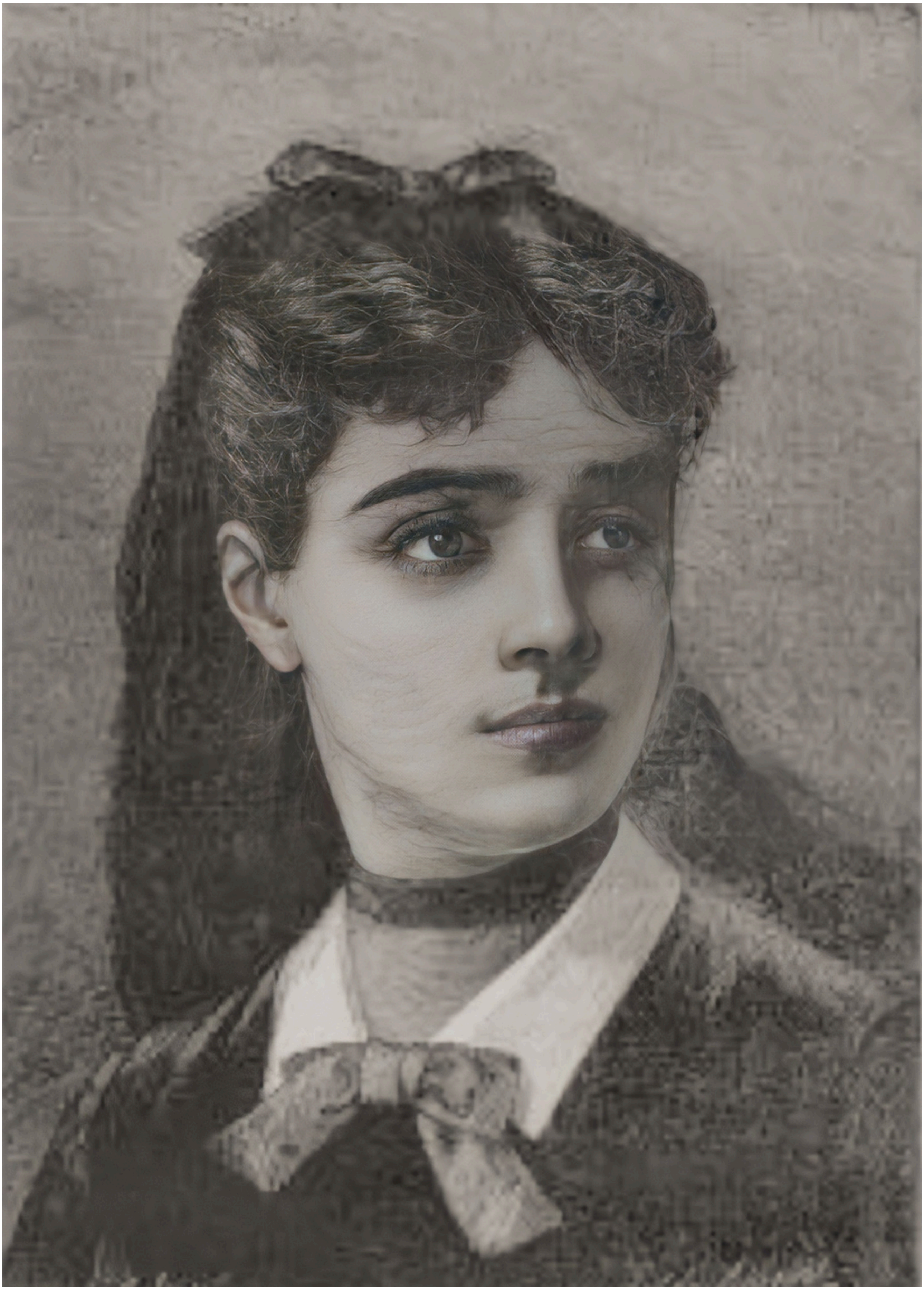
3.4 Death and Legacy

Agnesi's work had a lasting impact on the scientific community and mathematicians of subsequent generations. Her book “Analytical institutions” remained a prominent reference for students and scholars alike, serving as a cornerstone for the study of calculus.

Additionally, her emphasis on clear explanations and logical reasoning set a standard for mathematical writing, shaping the way mathematics is taught and communicated to this day.

How she died:

Maria Gaetana Agnesi died on 9^{th} January 1799. She passed away in Milan, Italy, at the age of 81. The exact cause of her death is not widely documented, but it is believed to have been related to natural causes due to her advanced age.



Marie-Sophie Germain (1776-1831)

○ **Chapter 4**

Marie Sophie Germaine

4.1 Life History

Just a decade before the outbreak on an ancient street Rue Saint Denis of French revolution *Marie Sophie Germaine* was born on the first of April 1776 in *Paris*, *France*. *Marie-Sophie* had one younger sister, *Angélique-Ambroise* (born 1779), and one elder sister, *Marie-Madeline* (born 29 May 1770).She was the middle daughter of *Ambroise- François Germaine*. Her mother was also named Marie-Madeline, and this plethora of "*Maries*" may have been the reason she went by Sophie. Her father Ambroise-François, a prosperous merchant, goldsmith and jeweller who later became a silk-merchant, and *Marie-Madeleine Gruguelu* - the daughter of the goldsmith *Jean Gruguelu* who was a friend of philosophers and political economists. *Ambroise-François,* was elected as deputy to the National Assembly in 1789 and later became a director of the Bank of France. Her elder sister *Marie-Madeleine Germain* married the notary *Charles Lherbette* in 1790 and they had one son *Armand-Jacques Lherbette* who became a lawyer, sportsman and politician. And her younger sister *Angélique-Ambroise Germain* married the doctor *Rene-Claude Geoffroy* in 1809 and, after his death in 1831; she married another medical man *Joachim-Henri Dutochet,* a leading *botanist* and *physiologist*, in 1833. We have given some details of Sophie's sisters because she did not marry and, as a consequence, her sisters and their families played a large part in her life.

She was a *French mathematician*, *physicist*, and *philosopher*. A story told of those years is that Sophie *Germain* read the story of *Archimedes of Syracuse* who was reading geometry as he was killed. She was moved by this story and decided that she too must become a mathematician. She told her parents that she wanted to become a mathematician but they were totally opposed to her ideas, telling her that it was no occupation for a girl. *Germain's* parents did not at all approve of her sudden fascination with mathematics, which was then thought inappropriate for a woman. When night came, they would deny her warm clothes and a fire for her bedroom to try to keep her from studying, but after they left, she would take out candles, wrap herself in quilts and do mathematics. Eventually her parents lessened their opposition to her studies, after waking up one morning, seeing Sophie was not in her bed, and finding her asleep in the library which was so cold that the ink had frozen solid in the ink well. Although *Germain* neither married nor obtained a professional position, her father continued to support her financially throughout her life. Being a female, she never wavered in her pursuit of mathematics despite the hindrances in a patriarchal society that barred a woman from taking education. In her early life, she kept hidden her identity and worked under the pseudonym of Leblanc. Mainly she taught herself by studying books from her father’s library. Her passion for mathematics despite obstacles and family pressure remain unrevealed, despite initial opposition from her parents and difficulties presented by society. She was thirteen years old when the Revolution broke out. She gained education from books in her father's library, including ones by Euler and from correspondence with famous mathematicians such as Lagrange, Legendre, and Gauss, under the pseudonym of Monsieur LeBlanc. Her work on Fermat's Last Theorem provided a foundation for mathematicians exploring the subject for hundreds of years after. Because of prejudice against her sex, she was unable to make a career out of mathematics, but she worked independently throughout her life. After some time, her mother even secretly supported her.

Germain thought that if the geometry method, which at that time referred to all of pure mathematics, could hold such fascination for Archimedes, it was a subject worthy of study. So she pored over every book on mathematics in her father's library, even teaching herself *Latin* and *Greek*, so she could read works like those of Sir *Isaac Newton* and *Leonhard Euler*. She continued to study the *differential* and *integral calculus* over the following years with the horrific events around her during the *Reign of Terror* in 1793-94 only helping her concentrate on her studies. *Germain* continued studying independently until she was 18, when she connected to a professor at a new and prestigious French University, *Ecole Polytechnique* (In 1794). During this time period, females were not allowed to attend lectures at the school, but *Germain* was determined to further her mathematical ability. She found a way to obtain lecture notes from many well-known mathematicians who taught there. She also submitted some of her own personal work to a professor named *Lagrange*, who had assigned homework that was recorded in his lecture notes. *Germain* submitted this homework under the male name *LeBlanc*, who had previously attended the school but passed away. Her submission caught the attention of *Lagrange*, who insisted on meeting the author. *Lagrange* then learned that *LeBlanc* was actually *Sophie Germain*, a female, and was greatly impressed by her. Lagrange did not mind that *Germain* was a woman and he became her mentor. He respected her ability and talent and brought her into his circle of *scientists* and *mathematicians*, an opportunity she had never gotten before. This motivated her even more in her mathematical research. Her relationship with *Lagrange* did not mark the end of *Germain's* correspondence with mathematicians under a hidden identity. In 1804, when she was 28 years old, *Germain* gained an interest in *number theory* and specifically *Fermat's Last Theorem*. She read both *Adrien-Marie Legendre's* and *Carl Gauss's* work regarding number theory and reached out to both of them. She started correspondence with *Gauss* about both his findings in number theory and the proof of *Fermat's Last Theorem*, again choosing to use *LeBlanc* name instead of revealing her identity as a woman. *Gauss* was impressed with the work of *LeBlanc* writing about "him" to his friend *Wilhelm*: "I am amazed that *M. LeBlanc* has completely mastered my *Disquisitiones Arithmeticae* and sent me very respectable communication about them". *Gauss* found out *Germain's* true identity three years into their correspondence and he responded to this surprise in the same way that *Lagrange* did– with adoration and great respect for her. He wrote about her again, saying "but when a woman, because of her sex, our customs and prejudices, encounters infinitely many more obstacles than men, in familiarizing herself with their knotty problems, yet overcomes these fetters and penetrates that which is most hidden, she doubtless has the most noble courage, extraordinary talent and superior genius".

In 1808, four years into their correspondence, *Gauss* took a new job as an astronomy professor and his wife passed away, so he adopted responding to *Germain's* letters. *Germain* then turned her attention to other fields of mathematics for a number of years, until a prize was offered for a correct proof of *Fermat's Last Theorem* in 1815. This motivated her to begin working on the plan she had begun to develop years earlier. In need of more experienced number theorists, she then began communication with *Legendre* as she continued working on the proof. In 1819 she wrote to *Gauss* one more time with the hope that he would respond. She shared with him her main idea to prove the *Fermat's Last Theorem* that showed how well she grasped some complex ideas in *Gauss's* work. Even though *Germain* made strides in proving *Fermat's Last Theorem* that eventually led other mathematicians to draw more conclusions with regard to the proof, she never chose to publish any of her efforts. Her work has long been known as a *footnote of Legendre's* publication about *Fermat's Last Theorem*, until a few mathematicians began to dig deeper into *Germain's* research in recent years. From these people, examining *Germain's* letters more carefully, we know that *Germain* had a very involved plan to prove *Fermat's Last Theorem*, proved case 1 of *Fermat's Last Theorem* for all odd primes less than or equal to 100, showed that any counter example to the Theorem would have to be least 30 digits long, and her theorem was later used to prove *Fermat's Last Theorem* for odd primes less than 1700.

Because of *Germain's* talent and love for mathematics, *Gauss* requested that she should be given an honorary degree from the *University of Gottingen*. Unfortunately, on 27 June 1831 she died in her house at the age of 55 due to breast cancer before she could receive it. Even though the ultimate goal of proving *Fermat's Last Theorem* was not fully accomplished by *Germain*, her work on this proof influenced other mathematicians as well as the study of number theory. Germain made strides in other fields besides mathematics as well. In 1831 *Crelle's* Journal published her paper on the *Curvature of Elastic Surfaces*. She also published in *Annales de chimie et de physique*- an examination of principles, which led to the discovery of the laws of equilibrium and movement of elastic solids.

Despite Germain's intellectual achievements, her death certificate lists her as a *property holder,* not a "mathematician". But her work was not unappreciated by everyone. When the matter of honorary degrees came up at the University of Gottingen in 1837—six years after Germain's death, Gauss lamented: "she [Germain] proved to the world that even a woman can accomplish something worthwhile in the most rigorous and abstract of the sciences and for that reason would well have deserved an honorary degree".

4.2 Contributions

Contributions to Mathematics

Mathematics has played a significant role in society due to the presence of mathematicians such as Sophie Germain. Sophie Germain was a self-educated mathematician who accomplished a great deal despite formidable obstacles preventing a woman of her time from pursuing a career in mathematics. This paper presents biographical information on Germain and her most significant work in number theory involves the proof for the famous Fermat's

Last Theorem (1637), a proof which was divided into two separate cases, the first case for all odd primes less than 100. Commonly known as “Sophie Germain’s Theorem,” it was later used by mathematician L.E. Dickson to prove Fermat’s Last Theorem for odd prime numbers that are less than 1700.In order for a prime number p to be considered a Sophie Germain prime, $2p +1$ must also be prime ($2p+1$ *is known as a safe prime*). *Germain’s* monumental work is discussed in her 1821 essay on the elasticity theory, which was published at her own expense. Her research was significantly based on mean curvature, a term coined by the famous French mathematician herself. The mean curvature is essential in the analysis of minimal surfaces as well as of the physical interfaces.

Germain first became interested in number theory in 1798 when Adrien-Marie Legendre published *Essai sur la théorie des nombres*. After studying the work, she opened correspondence with him on number theory, and later elasticity. Legendre included some of Germain's work in the Supplément to his second edition of the Théorie des Nombres, where he calls it très ingénieuse ("very ingenious").

Correspondence with Gauss:

Germain's interest in number theory was renewed when she read Carl Friedrich Gauss' monumental work *Disquisitiones Arithmeticae.* After three years of working through the exercises and trying her own proofs for some of the theorems, she wrote, again under the pseudonym of M. LeBlanc, to the author himself, who was one year younger to her. The first letter, dated on 21 November 1804, discussed Gauss' Disquisitiones and presented some of Germain's work on Fermat's Last Theorem. In the letter, Germain claimed to have proved the theorem for $n=p-1$, where p is a prime number of the form $p=8k+7$. However, her proof contained a weak assumption, and Gauss's reply did not comment on Germain's proof.

Around 1807 (sources differ), during the Napoleonic wars, the French were occupying the German town of Braunschweig, where Gauss lived. Germain, concerned that he might suffer the fate of Archimedes, wrote to General Pernety (Joseph Marie de Pernety), a family friend, requesting that he ensure Gauss' safety. General Pernety sent the chief of a battalion to meet with Gauss personally to see that he was safe. As it turned out, Gauss was fine, but he was confused by the mention of Sophie's name.

Renewed interest:

Germain's best work was in number theory and her most significant contribution to number theory dealt with Fermat's Last Theorem. In 1815, after the elasticity contest, the Academy offered a prize for a proof of Fermat's Last Theorem. It reawakened Germain's interest in number theory, and she wrote to Gauss again after ten years of no correspondence.

In the letter, Germain said that number theory was her preferred field and that it was in her mind all the time she was studying elasticity. She outlined a strategy for a general proof of Fermat's Last Theorem, including a proof for a special case. Germain's letter to Gauss contained her substantial progress toward a proof. She asked Gauss whether her approach to the theorem was worth pursuing. Gauss never answered.

When Germain's correspondence with Gauss ceased, she took interest in a contest sponsored by the Paris Academy of Sciences concerning Ernst Chladni's experiments with vibrating metal plates. The object of the competition, as stated by the Academy, was "to give the mathematical theory of the vibration of an elastic surface and to compare the theory to experimental evidence". Lagrange's comment that a solution to the problem would require the invention of a new branch of analysis deterred all but two contestants, Denis Poisson and Germain. Then Poisson was elected to the Academy, thus becoming a judge instead of a contestant, and leaving Germain as the only entrant to the competition. In 1809 Germain began work. Legendre assisted by giving her equations, references, and current research. She submitted her paper early in the fall of 1811 and did not win the prize. The judging commission felt that the true equations of the movement were not established, even though the experiments presented ingenious results. Lagrange was able to use Germain's work to derive an equation that was correct under special assumptions.

Other Contributions

Her other contributions are in the spheres of philosophy and sociology. Her philosophy regarding sociology influenced the later thinkers like August Comte. She classified the facts by generalizing them into laws as foundation of science of psychology and sociology. But her interest in elasticity remained an inspiration. As Mary Gray said “She also published in *Annales de chimie et de* physique an examination of principles which led to the discovery of the laws of equilibrium and movement of elastic solids.”

In recognition of her contribution towards advancement of mathematics, an honorary degree was also conferred upon her by university of Gottingen after six years of her death.

Recognition for Her Work

This famous essay was also her third attempt at winning a competition based on the scientific experiments conducted by German physicist Ernst Chladni and which was sponsored by the French Academy of Sciences. This work – deriving an accurate differential equation for the vibration of elastic surfaces – finally won her the coveted prize on January 8, 1816, and she also remains in history as the first female scientist to win a prize awarded by the famous French learned society. She was allowed to attend the Paris Academy of Science’s sessions fors seven years, later, after making friends with the Academy’s secretary, Joseph Fourier. In 1831, posthumously, another of Germain’s important papers on the elasticity theory and curvature was published by Crelle’s Journal.

Honours

Germain's resting place in the Pere La-chaise Cemetery in Paris is marked by a gravestone. At the centennial celebration of her life, a street and a girl’s school were named after her, and a plaque was placed at the house where she died. The school houses a bust commissioned by the Paris City Council. In January 2020, Satellogic, a high-resolution Earth observation imaging and analytic company, launched a ÑuSat type micro-satellite named in honour of Sophie Germain.

4.3 Prizes and Awards

On January 18, 1816 she won the prestigious Paris academy of science prize. She was the first woman to do so. Its purpose was to honour a French mathematician for research in the foundations of mathematics. This award, in the amount of €8,000, was established in 2003, under the auspices of the *Institut de France.*

Quotes by Germain

- "Algebra is nothing more than geometry, in other words geometry is nothing more than algebra in pictures."
  Sophie Germain

- "It matters little who first arrived at an idea, rather than what is significant is how far that idea can go."
  Sophie Germain

4.4 Death and Legacy

Mentored by the famous mathematician Joseph-Louis Lagrange, who supported her both morally and professionally, Sophie Germain passionately and devotedly continued her work in mathematics and philosophy until her death.

Sophie Germain passed away at the age of 55 on June 27, 1831, in Paris, France and she was buried in the Pere La-chaise Cemetery. She had been suffering from breast cancer since 1829. On her death certificate, she was listed as a property holder.

Sofya Kovalevskaya (1850-1891)

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Chapter 5

5.1 Life History

Sofya Kovalevskaya was born in Moscow on 15th of January 1850, the second of three children. Sofya Kovalevskaya was a Russian mathematician who made noteworthy contributions to analysis, partial differential equations and mechanics. She was a pioneer for women in mathematics around the world – the first woman to obtain a doctorate (in the modern sense) in mathematics, the first woman appointed to a full professorship in northern Europe and one of the first women to work for a scientific journal as an editor. According to historian of science Ann Hibner Koblitz, Kovalevskaya was "the greatest known woman scientist before the twentieth century". Her father, Lieutenant General Vasily Vasilyevich Korvin-Krukovsky, served in the Imperial Russian Army as head of the Moscow Artillery before retiring to Polibino, his family estate in Pskov Oblast in 1858, when Kovalevskaya was eight years old. He was a member of the minor nobility, of mixed (Bela) Russian–Polish descent (Polish on his father's side), with possible partial ancestry from the royal Corvin family of Hungary and served as Marshall of Nobility for Vitebsk province. Her mother, Yelizaveta Fedorovna Shubert (Schubert), descended from a family of German immigrants to St. Petersburg who lived on Vasilievsky Island. Her maternal great-grandfather was the astronomer and geographer Friedrich Theodor Schubert, who emigrated to Russia from Germany around 1785. Kovalevskaya's parents provided her with a good early education. At various times, her governesses were native speakers of English, French, and German. When she was 11 years old, she was intrigued by a foretaste of what she was to learn later in her lessons in calculus; the wall of her room had been papered with pages from lecture notes by Ostrogradsky, left over from her father's student days. She was tutored privately in elementary mathematics by Iosif Ignatevich Malevich.

The physicist Nikolai Nikanorovich Tyrtov noted her unusual aptitude when she managed to understand his textbook by discovering for herself an approximate construction of trigonometric functions which she had not yet encountered in her studies. Tyrtov called her a "New Pascal" and suggested she be given a chance to pursue further studies under the tutelage of N. Strannoliubskii. In 1866-67 she spent much of the winter with her family in St. Petersburg, where she was provided private tutoring by Strannoliubskii, a well-known advocate of higher education for women, who taught her calculus. During the same period, the son of a local priest introduced her sister Anna to progressive ideas influenced by the radical movement of the 1860s, providing her with copies of radical journals of the time discussing Russian nihilism. Despite her obvious talent for mathematics, she could not complete her education in Russia. At that time, women were not allowed to attend universities in Russia and most other countries. In order to study abroad, Kovalevskaya needed written permission from her father (or husband). Accordingly, in 1868 she contracted a "fictitious marriage" with Vladimir Kovalevskij, a young paleontology student, book publisher and radical, who was the first to translate and publish the works of Charles Darwin in Russia. They moved from Russia to Germany in 1869, after a brief stay in Vienna, in order to pursue advanced studies. In April 1869, she attended lectures in physics at the university, they moved to Heidelberg. Through great efforts, she obtained permission to audit classes with the professors' approval at the University of Heidelberg. There she attended courses in physics and mathematics under teachers such as Hermann von Helmholtz, Gustav Kirchhoff and Robert Bunsen. In October 1869, shortly after attending courses in Heidelberg, she visited London with Vladimir, who spent time with his colleagues Thomas Huxley and Charles Darwin, while she was invited to attend George Eliot's Sunday salons. There, at age nineteen, she met Herbert Spencer and was led into a debate, at Eliot's instigation, on "woman's capacity for abstract thought". Although there is no record of the details of their conversation, she had just completed a lecture course in Heidelberg on mechanics, and she may just possibly have made mention of the Euler equations governing the motion of a rigid body.

In October 1870, Kovalevskaya moved to Berlin, where she began to take private lessons with Karl Weierstrass, since the university would not allow her even to audit classes. He was very impressed with her mathematical skills and over the subsequent three years taught her the same material that comprised his lectures at the university. In 1871 she briefly traveled to Paris together with Vladimir in order to help in the Paris Commune, where Kovalevskaya attended the injured and her sister Anyuta was active in the Commune. Kovalevskaya returned to Berlin and continued her studies with Weierstrass for three more years. In 1874 she presented three papers—on partial differential equations, on the dynamics of Saturn's rings, and on elliptic integrals—to the University of Göttingen as her doctoral dissertation. With the support of Weierstrass, this earned her a doctorate in mathematics summa cum laude, after Weierstrass succeeded in having her exempted from the usual oral examinations.

In 1884 Kovalevskaya was appointed to a five-year position as Extraordinary Professor (assistant professor in modern terminology) and became an editor of Acta Mathematica. In 1888 she won the Prix Bordin of the French Academy of Science, for her work on the rotation of a solid body around a fixed point. Her submission featured the celebrated discovery of what is now known as the "Kovalevskaya top", which was subsequently shown to be the only other case of rigid body motion that is "completely integrable" other than the tops of Euler and Lagrange.

In 1889 Kovalevskaya was appointed Ordinary Professor (full professor) at Stockholm University, the first woman in Europe in modern times to hold such a position. After much lobbying on her behalf (and a change in the Academy's rules) she was made a Corresponding Member of the Russian Academy of Sciences, but she was never offered a professorship in Russia.

Kovalevskaya thereby became the first woman to have been awarded a doctorate (in the modern sense of the world) in mathematics. Her paper on partial differential equations contains what is now commonly known as the Cauchy–Kovalevskaya theorem, which proves the existence and analyticity of local solutions to such equations under suitably defined initial/boundary conditions.

5.2 Contributions

Cauchy Problem

The Cauchy problem is currently referred to as the "Cauchy-Kovalevskaya theorem" (Morrow & Perl, 1998). She described the application of this theory in her dissertation which was titled, "on the theory of partial differential equations" (Morrow & Perl, 1998). She developed this theory to help in solving a system of general differential equations. The differential equations in this case were of the first order and could be used with any variable. Her study of deferential equations was informed by Weierstrass' concept on total equations. Her contribution in the study of differential equations is found on the fact that she transformed Weierstrass' total equations into "partial differential equations" (Morrow & Perl, 1998). Today, Kovalevskaya's concept is used to solve differential equations which have initial conditions (Cauchy problem). The theory is useful in solving hyperbolic equations. However, there is a lot of difficulty in using the theory to solve elliptic and parabolic equations.

Abelian Integrals

Kovalevskaya's study of the concept of Abelian integrals is found in her work titled, "on the reduction of a definite class of Abelian integrals of the third range" (Morrow & Perl, 1998). Building on Weierstrass' theory of Abelian integrals, she developed a series of skilled manipulations which explained the application of the theory. Thus her main contribution involved using her understanding to illustrate how the concept of Abelian integrals can be used to solve various mathematical equations.

Euler's Equations

Kovalevskaya's contributions to the development of Euler's equations are explained in a publication titled, "on the property of a system of equations" (Morrow & Perl, 1998). Euler's equations help in studying the motion of rigid bodies that are rotating towards a given direction. The equations are made up of six differential equations of the first order. This system of differential equations is associated with great symmetry. Even though Euler's equations had great symmetry, Euler did not succeed in finding their solutions. Drawing from the concept of theta equations, Kovalevskaya explained how Euler's equations can be solved by using algebraic integrals and variable transformation. She completed the investigations of Lagrange and Euler by illustrating how the Euler's equations can be used to solve equations that are associated with motion. She also developed the concept of "Kovalevskaya's top" (Morrow & Perl, 1998) by studying movable poles in order to understand the integration that is associated with a dynamical system. In this case, she helped scientist to realize the importance of complex analysis in solving mathematical equations.

Bruns' Theorem

Kovalevskaya's study of the Bruns' theorem is illustrated in her work that was titled, "sur un theorem de M. Bruns" (Morrow & Perl, 1998). Her contribution in this case involved developing a simpler approach for proving Bruns' theorem. She helped in proving that Bruns' theorem is "a function of a homogeneous body" (Morrow & Perl, 1998).

5.3 Conclusion

Her research and contributions were based on the studies that had been done earlier by other mathematicians and scientists. The concepts and theories that she developed are still used today to solve various mathematical equations. Her study of the concept of Abelian integrals helped in explaining how the concept can be used to solve various mathematical equations.

5.4 Death and legacy

Kovalevskaya died of epidemic influenza complicated by pneumonia in 1891 at age forty-one, after returning from a vacation in Nice with Maxim. She is buried in Solna, Sweden, at Norra begravningsplatsen. Sonya Kovalevsky High School Mathematics Day is a grant-making program of the Association for Women in Mathematics (AWM), funding workshops across the United States which encourages girls to explore mathematics. The Kovalevsky Lecture is sponsored annually by the AWM and the Society for Industrial and Applied Mathematics and is intended to highlight significant contributions of women in the fields of applied or computational mathematics.

Amalie Emmy Neother (1882-1935)

Chapter 6

Amalie Emmy Noether

6.1 Life History

Amalie Emmy Noether was a German mathematician born to a Jewish family in the small city of Erlangen in Franconia Germany on March 23, 1882. Her father, Max Noether, was an eminent professor of mathematics at the University of Erlangen. Her mother was Ida Amalia Kaufmann, whose family were wealthy wholesalers. Young Emmy was brought up as a typical girl of her era: helping with cooking and running the house – she admitted later she had little aptitude for these sort of things. Her mother was a skilled pianist, but Emmy did not enjoy piano lessons. Her main passion was dancing. She also loved mathematics, but she knew that the rules of German society meant she would not be allowed to follow in her father's footsteps to become a university academician. After completing high school – she attended the Municipal School for Higher Education of Daughters in Erlangen – she trained to become a school teacher, qualifying in 1900, aged 18, to teach English and French in girls' schools.

Although a career in teaching offered her financial security, her love of mathematics proved to be too strong. She decided to abandon teaching and apply to the University of Erlangen to observe mathematics lectures there. She could only observe lectures, because women were not permitted to enroll officially at the university. Between 1900 and 1902 Emmy studied mathematics at Erlangen. In July 1903 she traveled to the city of Nurnberg and passed the matriculation examination allowing her to study mathematics (but not officially enrolled) at any German university. Emmy chose to go for a semester to the University of Gottingen ,back then home to the most prestigious school of mathematics in the world. Some of the greatest mathematicians in history had taught at Gottingen, including Carl Friedrich Gauss and Bernhard Riemann. Emmy attended lectures given by: Hermann Murkowski, the esteemed mathematician who taught Albert Einstein, and David Hilbert, probably the twentieth century's most outstanding mathematician.

In 1904 Emmy was overjoyed to learn that her hometown university, Erlangen, had decided women should be permitted full access. She was accepted as a Ph.D. student by the renowned mathematician Paul Gordan, who was 67 when Emmy started to work with him. She was the only student he ever accepted as a Ph.D. candidate. Gordan was known among mathematicians as "the king of invariant theory." Emmy made exceptional progress in this field, which would later lead to her making a remarkable discovery in physics. In 1907 the 25-year-old Emmy officially became Doctor Noether. Her degree was awarded 'summa cum laude" – the highest distinction possible. In 1908 Noether was appointed to the position of mathematics lecturer at Erlangen. Unfortunately, it was an unpaid position. This was not especially unusual in Germany for a first lecturing job. The great chemist Robert Bunsen's first lecturing position was without pay at the University of Göttingen.

David Hilbert familiarized himself with Noether's research; like her father, he recognized her outstanding ability. By this stage in his career Hilbert was concerned mainly with physics, which he believed needed help from the best mathematicians. In 1913 and 1914 Noether exchanged letters with David Hilbert and his Göttingen colleague Felix Klein discussing Einstein's Relativity Theory.

In 1915 Hilbert invited her to become a lecturer in Göttingen. Unfortunately, this provoked a storm of protest from the history and linguistics faculties who did not think it appropriate that a woman should be teaching men, particularly since Germany was at war during the period– World War 1: 1914 – 1918. Although in general the mathematics and science faculties supported Noether, they could not overcome the opposition from the humanities. Noether was so eager to join Hilbert's department in Göttingen that, to soothe Hilbert's opponents, she agreed not to be formally appointed as a lecturer and to receive no pay. Her father continued supporting her financially (sadly her mother died in 1915) and the lectures she gave were advertised as lectures by Professor Hilbert, with assistance from Dr. E. Noether.

In the winter of 1928–1929 Noether accepted an invitation to Moscow State University, where she continued working with P.S. Alexandrov. In addition to carrying on with her research, she taught classes in abstract algebra and algebraic geometry. She worked with the topologists Lev Pontryagin and Nikolai Chebotaryov, who later praised her contributions to the development of Galois Theory. Noether taught at the Moscow State University during the winter of 1928–1929.

Although politics was not central to her life, Noether took a keen interest in political matters and according to Alexandrov, showed considerable support for the Russian Revolution. She was especially happy to see Soviet advances in the fields of science and mathematics, which she considered indicative of new opportunities made possible by the Bolshevik project. This attitude caused her problems in Germany. Noether planned to return to Moscow, an effort for which she received support from Alexandrov. After she left Germany in 1933 he tried to help her gain a chair at Moscow State University through the Soviet Education Ministry. Although this effort proved unsuccessful, they corresponded frequently during the 1930s, and in 1935 she made plans for a return to the Soviet Union. Meanwhile, her brother Fritz accepted a position at the Research Institute for Mathematics and Mechanics in Tomsk, in the Siberian Federal District of Russia, after losing his job in Germany and was subsequently executed during the Great terror.

Noether's parents supported her as much as they could through this time, her father recognizing something rather special in his daughter's capabilities. Nevertheless, her life was a struggle financially. While working as a lecturer, Noether became fascinated by the work David Hilbert had done in Göttingen. The work was more abstract than any she had done at Erlangen. She began stretching and modifying Hilbert's methods. This was her first heavyweight encounter with abstract algebra, mathematical territory in which she would soon become a powerful innovator.

Noether was totally devoted to mathematics and talked little of anything else. She never married and had no children. She cared little for her appearance and less for social conventions; she was not a shrinking violet – she spoke loudly and forcibly. She could be very blunt when she disagreed with anyone on a mathematical issue, and people with whom she disagreed could feel rather bruised mentally. On the other hand, she was very kind, considerate, and unselfish with everyone, and would go out of her way to ensure her Ph.D. students got full credit for their work, even when she had contributed significantly to it herself. Only students who were very bright and fully prepared benefited from her rather disorganized lectures – like Willard Gibbs's students. To her advanced students, she would present ideas at the forefront of modern mathematics – concepts that she herself was currently working on. This was of great benefit to her best students, who were able to publish research papers based on new, entirely original ideas Noether had been discussing in her lectures.

Her best lessons were delivered informally, in conversations, or when walking out with her students, for whom she always had time. Her students were sometimes called the "Noether Boys".

In 1932 The Ackermann–Teubner Memorial Award, which was a prize given by the Leipzig publishing house B. G. Teubner to mathematicians who made significant contributions to the advancement of mathematical sciences was bestowed on her. Noether shared the award with Emil Artin, her former student and colleague. Noether's colleagues celebrated her fiftieth birthday in 1932, in typical mathematicians' style. Helmut Hasse dedicated an article to her in the Mathematische Annalen, wherein he confirmed her suspicion that some aspects of noncommutative algebra are simpler than those of commutative algebra, by proving a noncommutative reciprocity law. This pleased her immensely. He also sent her a mathematical riddle, which he called the "mμν-riddle of syllables". She solved it immediately, but the riddle has been lost.

In 1932 September, Noether delivered a plenary address (groberVortrag) on "Hyper-complex systems in their relations to commutative algebra and to number theory" at the International Congress of Mathematicians in Zürich. The congress was attended by 800 people, including Noether's colleagues Hermann Weyl, Edmund Landau, and Wolfgang Krull. There were 420 official participants and twenty-one plenary addresses presented. Apparently, Noether's prominent speaking position was recognition of the importance of her contributions to mathematics.

The Emmy Noether Lectureship by the Association for Women in Mathematics since 1980, which is a series of biennial lectures given by prominent women mathematicians at the Joint Mathematics Meetings. The lectureship is named after Noether to honor her legacy and achievements in mathematics. The Emmy Noether Awards are given by The Brown Foundation since 2019, which are prizes given to outstanding female undergraduate students who aspire to pursue careers in STEM fields. The awards provide financial and networking assistance to the winners, who are selected based on their academic excellence, research potential, and leadership skills. The ICM Emmy Noether Lecture by the International Mathematical Union since 1994, which is a special lecture given by a distinguished woman mathematician at the International Congress of Mathematicians. The lecture aims to highlight the achievements and contributions of women in mathematics and to inspire future generations of female mathematicians.

When Adolf Hitler became the German Reichskanzler in January 1933, Nazi activity around the country increased dramatically. At the University of Göttingen the German Student Association led the attack on the "UnGerman spirit" attributed to Jews and was aided by a privatdozent named Werner Weber, a former student of Noether. Anti-Semitic attitudes created a climate hostile to Jewish professors. One young protester reportedly demanded: "Aryan students want Aryan mathematics and not Jewish mathematics. One of the first actions of Hitler's administration was the Law for the Restoration of the Professional Civil Service which removed Jews and politically suspected government employees (including university professors) from their jobs. Noether accepted the decision calmly, providing support for others during this difficult time. Noether was contacted by representatives of two educational institutions: Bryn Mawr College, in the United States, and Somerville College at the University of Oxford, in England. After a series of negotiations with the Rockefeller Foundation, a grant to Bryn Mawr was approved for Noether and she took a position there, starting in late 1933. In 1934, Noether began lecturing at the Institute for Advanced Study in Princeton upon the invitation of Abraham Flexner and Oswald Veblen. She also worked with and supervised Abraham Albert and Harry Vandiver. Emmy Noether died in Bryn Mawr at the age of 53 on April 14, 1935. She died of complications a few days after an operation to remove two smaller tumors from her pelvis. The cause of death was possibly a viral infection. A few days after Noether's death her friends and associates at Bryn Mawr held a small memorial service at College President Park's house. Her body was cremated and her ashes were buried under the cloisters of Bryn Mawr College’s M. Carey Thomas Library.

6.2 Contributions

Noether's work in abstract algebra and topology was influential in mathematics, while in physics, Noether's theorem has consequences for theoretical physics and dynamical systems. She showed an acute propensity for abstract thought, which allowed her to approach problems of mathematics in fresh and original ways. In the first epoch (1907–1919), Noether dealt primarily with differential and algebraic invariants, beginning with her dissertation under Paul Gordan. Her mathematical horizons broadened, and her work became more general and abstract, as she became acquainted with the work of David Hilbert, through close interactions with a successor to Gordan, Ernst Sigismund Fischer.

After moving to Göttingen in 1915, she produced her work for physics, the two Noether's theorems. In the second epoch (1920–1926), Noether devoted herself to developing the theory of mathematical rings. In the third epoch (1927–1935), Noether focused on noncommutative algebra, linear transformations, and commutative number fields.

Although the results of Noether's first epoch were impressive and useful, her fame among mathematicians rests more on the groundbreaking work she did in her second and third epochs, as noted by Hermann Weyl and B.L. van der Waerden in their obituaries of her.

First Epoch:

Much of Noether's work in the first epoch of her career was associated with invariant theory, principally algebraic invariant theory. Invariant theory is concerned with expressions that remain constant (invariant) under a group of transformations. As an everyday example, if a rigid yardstick is rotated, the coordinates (x_1, y_1, z_1) and (x_2, y_2, z_2) of its endpoints change, but its length L given by the formula $L^2 = \Delta x^2 + \Delta y^2 + \Delta z^2$ remains the same. Invariant theory was an active area of research in the later nineteenth century, prompted in part by Felix Klein's Erlangen program, according to which different types of geometry should be characterized by their invariants under transformations, e.g., the cross ratio of projective geometry.

An example of an invariant is the discriminant $B^2 - 4AC$ of a binary quadratic form $\mathbf{x}\cdot\mathrm{A}\,\mathbf{x} + \mathbf{y}\cdot\mathrm{B}\,\mathrm{x} + \mathbf{y}\cdot\mathrm{C}\,\mathbf{y}$, where $\mathbf{x}$ and $\mathbf{y}$ are vectors and "·" is the dot product or "inner product" for the vectors. A, B, and C are linear operators on the vectors – typically matrices. The discriminant is called "invariant" because it is not changed by linear substitutions $\boldsymbol{x} \rightarrow a\boldsymbol{x} + b\boldsymbol{y}$, $y \rightarrow c\boldsymbol{x} + d\boldsymbol{y}$ with determinant $ad - bc = 1$. These substitutions form the special linear group SL_2.

One can ask for all polynomials in A, B, and C that are unchanged by the action of SL_2; these are called the invariants of binary quadratic forms and turn out to be the polynomials in the discriminant. More generally, one can ask for the invariants of homogeneous polynomials $A_0 x^r y^0 + ... + A_r x^0 y^r$ of higher degree, which will be certain polynomials in the coefficients $A_0, ..., A_r$, and more generally still, one can ask the similar question for homogeneous polynomials in more than two variables.

One of the main goals of invariant theory was to solve the "finite basis problem". The sum or product of any two invariants is invariant, and the finite basis problem asked whether it was possible to get all the invariants by starting with a finite list of invariants, called generators, and then, adding or multiplying the generators together. For example, the discriminant gives a finite basis (with one element) for the invariants of binary quadratic forms. Noether's advisor, Paul Gordan, was known as the "king of invariant theory", and his chief contribution to mathematics was his 1870 solution of the finite basis problem for invariants of homogeneous polynomials in two variables. He proved this by giving a constructive method for finding all of the invariants and their generators, but was not able to carry out this constructive approach for invariants in three or more variables. In 1890, David Hilbert proved a similar statement for the invariants of homogeneous polynomials in any number of variables. Furthermore, his method worked, not only for the special linear group, but also for some of its subgroups such as the special orthogonal group.

Galois Theory

Galois Theory concerns transformations of number fields that permute the roots of an equation. Consider a polynomial equation of a variable x of degree n, in which the coefficients are drawn from some ground field, which might be, for example, the field of real numbers, rational numbers, or the integers modulo 7. There may or may not be choices of x, which make this polynomial evaluate to zero. Such choices, if they exist, are called roots. If the polynomial is $x^2 + 1$ and the field is the real numbers, then the polynomial has no roots, because any choice of x makes the polynomial greater than or equal to one. If the field is extended, however, then the polynomial may gain roots, and if it is extended enough, then it always has a number of roots equal to its degree.

Continuing the previous example, if the field is enlarged to the complex numbers, then the polynomial gains two roots, $+i$ and $-i$, where i is the imaginary unit, that is, $i^2 = -1$. More generally, the extension field in which a polynomial can be factored into its roots is known as the splitting field of the polynomial.

The Galois group of a polynomial is the set of all transformations of the splitting field which preserve the ground field and the roots of the polynomial. (In mathematical jargon, these transformations are called automorphisms.) The Galois group of $x^2 + 1$ consists of two elements: The identity transformation, which sends every complex number to itself, and complex conjugation, which sends $+i$ to $-i$. Since the Galois group does not change the ground field, it leaves the coefficients of the polynomial unchanged, so it must leave the set of all roots unchanged. Each root can move to another root, however, such transformation determines a permutation of the n roots among themselves. The significance of the Galois group derives from the fundamental theorem of Galois Theory, which proves that the fields lying between the ground field and the splitting field are in one-to-one correspondence with the subgroups of the Galois group. In 1918, Noether published a paper on the inverse Galois problem. Instead of determining the Galois group of transformations of a given field and its extension, Noether asked whether, given a field and a group, it always is possible to find an extension of the field that has the given group as its Galois group. She reduced this to "Noether's problem", which asks whether the fixed field of a subgroup G of the permutation group S_n acting on the field $k(x_1,..., x_n)$ always is a pure transcendental extension of the field k. (She first mentioned this problem in a 1913 paper, where she attributed the problem to her colleague Fischer.) She showed this was true for $n = 2, 3$, or 4. In 1969, R.G. Swan found a counter-example to Noether's problem, with $n = 47$ and G a cyclic group of order 47 (although this group can be realized as a Galois group over the rationals in other ways). The inverse Galois problem remains unsolved.

Second Epoch:

Noether became famous for her deft use of ascending or descending chain conditions. A sequence of non-empty subsets A_1, A_2, A_3, etc. of a set S is usually said to be ascending, if each is a subset of the next

$$A_1 \subset A_2 \subset A_3 \subset \cdots.$$

Conversely, a sequence of subsets of S is called *descending* if each contains the next subset:

$$A_1 \supset A_2 \supset A_3 \supset \cdots.$$

A chain *becomes constant after a finite number of steps* if there is an n such that $A_n = A_m$ for all $m \geq n$. A collection of subsets of a given set satisfies the ascending chain condition if any ascending sequence becomes constant after a finite number of steps. It satisfies the descending chain condition if any descending sequence becomes constant after a finite number of steps.

Ascending and descending chain conditions are general, meaning that they can be applied to many types of mathematical objects-and, on the surface, they might not seem very powerful. Noether showed how to exploit such conditions, however, to maximum advantage.

For example: How to use chain conditions to show that every set of sub-objects has a maximal/minimal element or that a complex object can be generated by a smaller number of elements. These conclusions often are crucial steps in a proof. Many types of objects in abstract algebra can satisfy chain conditions, and usually if they satisfy an ascending chain condition, they are called Noetherian in her honor. By definition, a Noetherian ring satisfies an ascending chain condition on its left and right ideals, whereas a Noetherian group is defined as a group in which every strictly ascending chain of subgroups is finite. A Noetherian module is a module in which every strictly ascending chain of submodules becomes constant after a finite number of steps. A Noetherian space is a topological space in which every strictly ascending chain of open subspaces becomes constant after a finite number of steps; this definition makes the spectrum of a Noetherian ring a Noetherian topological space.

The chain condition often is "inherited" by sub-objects. For example, all subspaces of a Noetherian space are Noetherian themselves; all subgroups and quotient groups of a Noetherian group are likewise, Noetherian; and, mutatis mutandis, the same holds for submodules and quotient modules of a Noetherian module. All quotient rings of a Noetherian ring are Noetherian, but that does not necessarily hold for its subrings. The chain condition also may be inherited by combinations or extensions of a Noetherian object. For example, finite direct sums of Noetherian rings are Noetherian, as is the ring of formal power series over a Noetherian ring. Another application of such chain conditions is in Noetherian induction—also known as well-founded induction— which is a generalization of mathematical induction. It frequently is used to reduce general statements about collections of objects to statements about specific objects in that collection. Suppose that *S* is a partially ordered set. One way of proving a statement about the objects of *S* is to assume the existence of a counterexample and deduce a contradiction, thereby proving the contra positive of the original statement. The basic premise of Noetherian induction is that every non-empty subset of *S* contains a minimal element. In particular, the set of all counter examples contains a minimal element, the minimal counter example. In order to prove the original statement, therefore, it suffices to prove something seemingly much weaker.

Commutative rings, ideals, and modules

Noether's paper, *Idealtheorie in Ringbereichen* (Theory of Ideals in Ring Domains, 1921), is the foundation of general commutative ring theory and gives one of the first general definitions of a commutative ring. Before her paper, most results in commutative algebra were restricted to special examples of commutative rings, such as polynomial rings over fields or rings of algebraic integers. Noether proved that in a ring which satisfies the ascending chain condition on ideals, every ideal is finitely generated. In 1943, French mathematician Claude Chevalley coined the term, Noetherian ring, to describe this property. A major result in Noether's 1921 paper is the Lasker–Noether theorem, which extends Lasker's theorem on the primary decomposition of ideals of polynomial rings to all Noetherian rings. The Lasker–Noether theorem can be viewed as a generalization of the fundamental theorem of arithmetic which states that "any positive integer can be expressed as a product of prime numbers and that this decomposition is unique."

Noether's work Abstrakter Aufbau der Idealtheorie in algebraischen Zahl- und Funktionenkörpern (Abstract Structure of the Theory of Ideals in Algebraic Number and Function Fields, 1927) characterized the rings in which the ideals have unique factorization into prime ideals as the Dedekind domains: integral domains that are Noetherian, 0- or 1-dimensional and integrally closed in their quotient fields. This paper also contains what now are called the isomorphism theorems, which describe some fundamental natural isomorphisms and some other basic results on Noetherian and Artinian modules.

Elimination theory

In 1923–1924, Noether applied her ideal theory to elimination theory in a formulation that she attributed to her student, Kurt Hentzelt. She showed that fundamental theorems about the factorization of polynomials could be carried over directly. Traditionally, elimination theory is concerned with eliminating one or more variables from a system of polynomial equations, usually by the method of resultants. For illustration, a system of equations often can be written in the form $\mathrm{M}\,\mathbf{v} = \mathbf{0}$ where a matrix (or linear transform) M (without the variable x) times a vector $\mathbf{v}$ (that only has non-zero powers of x) is equal to the zero vector, $\mathbf{0}$. Hence, the determinant of the matrix M must be zero, providing a new equation in which the variable x has been eliminated.

Invariant theory of finite groups

Techniques such as Hilbert's original non-constructive solution to the finite basis problem could not be used to get quantitative information about the invariants of a group action, and furthermore, they did not apply to all group actions. In her 1915 paper, Noether found a solution to the finite basis problem for a finite group of transformations G acting on a finite-dimensional vector space over a field of characteristic zero. Her solution shows that the ring of invariants is generated by homogeneous invariants whose degree is less than, or equal to, the order of the finite group; this is called Noether's bound. Her paper gave two proofs of Noether's bound, both of which also work when the characteristic of the field is co prime to $|G|!$ (the factorial of the order $|G|$ of the group G). The degrees of generators need not satisfy Noether's bound when the characteristic of the field divides the number $|G|$ but Noether was not able to determine whether this bound was correct when the characteristic of the field divides $|G|!$ but not $|G|$. For many years, determining the truth or falsehood of this bound for this particular case was an open problem, called "Noether's gap". It was finally solved independently by Fleischmann in 2000 and Fogarty in 2001, who both showed that the bound remains true. In her 1926 paper, Noether extended Hilbert's theorem to representations of a finite group over any field; the new case that did not follow from Hilbert's work is when the characteristic of the field divides the order of the group. Noether's result was later extended by William Haboush to all reductive groups by his proof of the Mumford conjecture. In this paper Noether also introduced the Noether normalization lemma, showing that a finitely generated domain A over a field k has a set $\{x_1, ..., x_n\}$ of algebraically independent elements such that A is integral over $k[x_1, ..., x_n]$.

Contributions to topology

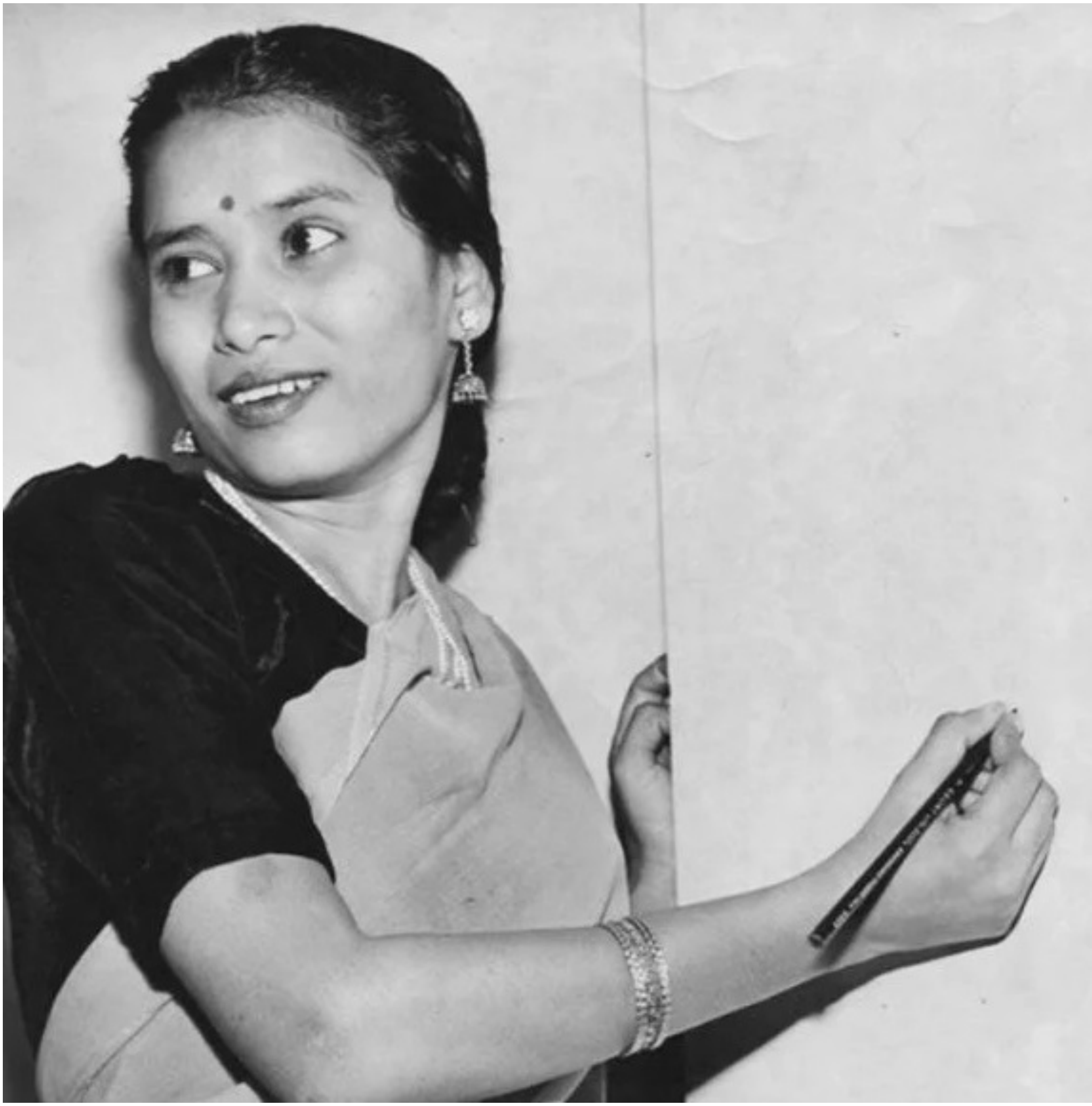
Noether's contributions to topology illustrate her generosity with ideas and how her insights could transform entire fields of mathematics. In topology, mathematicians study the properties of objects that remain invariant even under deformation, properties such as their connectedness. An old joke is that "a topologist cannot distinguish a donut from a coffee mug", since they can be continuously deformed into one another. Noether is credited with fundamental ideas that led to the development of algebraic topology from the earlier combinatorial topology, specifically, the idea of homology groups. According to the account of Alexandrov, Noether attended lectures given by Heinz Hopf and by him in the summers of 1926 and 1927, where "she continually made observations which were often deep and subtle and he continues that, when, she first became acquainted with a systematic construction of combinatorial topology, she immediately observed that it would be worthwhile to study directly the groups of algebraic complexes and cycles of a given polyhedron and the subgroup of the cycle group consisting of cycles homologous to zero; instead of the usual definition of Bette numbers, she suggested immediately defining the Bette group as the complementary (quotient) group of the group of all cycles by the subgroup of cycles homologous to zero. This observation now seems self-evident. But in those years (1925–1928) this was a completely new point of view. Noether's suggestion that topology be studied algebraically was adopted immediately by Hopf, Alexandrov and others, and it became a frequent topic of discussion among the mathematicians of Gottingen. Noether observed that her idea of a Betti group makes the Euler– Poincare formula simpler to understand and Hopf's own work on this subject "bears the imprint of these remarks of Emmy Noether". Noether mentions her own topology ideas only as an aside in a 1926 publication, where she cites it as an application of group theory. This algebraic approach to topology was also developed independently in Austria. In a 1926–1927 course given in Vienna, Leopold Vietoris defined a homology group, which was developed by Walther Mayer, into an axiomatic definition in 1928.

Third Epoch:

Noether did much work on hyper complex numbers and group representations was carried out in the nineteenth and early twentieth centuries, but remained disparate. Noether united these results and gave the first general representation theory of groups and algebras. Briefly, Noether subsumed the structure theory of associative algebras and the representation theory of groups into a single arithmetic theory of modules and ideals in rings satisfying ascending chain conditions. This single work by Noether was of fundamental importance for the development of modern algebra.

Noncommutative Algebra

Noether also was responsible for a number of other advances in the field of algebra. With Emil Artin, Richard Brauer, and Helmut Hasse, she founded the theory of central simple algebras. A paper by Noether, Helmut Hasse, and Richard Brauer pertains to division algebras, which are algebraic systems in which division is possible. They proved two important theorems: a local-global theorem stating that “if a finite dimensional central division algebra over a number field splits locally everywhere then it splits globally” (so is trivial) and from this, deduced their Hauptsatz ("main theorem"): *every finite dimensional central division algebra over an algebraic number field F splits over a cyclic cyclotomic extension.* These theorems allow one to classify all finite-dimensional central division algebras over a given number field. A subsequent paper by Noether showed, as a special case of a more general theorem, that all maximal subfields of a division algebra *D* are splitting fields. This paper also contains the Skolem–Noether theorem which states that any two embeddings of an extension of a field *k* into a finite-dimensional central simple algebra over k, are conjugate. The Brauer–Noether theorem gives a characterization of the splitting fields of a central division algebra over a field.



Shakuntala Devi (1929-2013)

Chapter 7

Shakuntala Devi

7.1 Life History

Shakuntala Devi was an Indian mathematician. She was born on Nov. 04 1929 in Banglore Karnataka to a Kannada Brahamin family. From a young age, Shakuntala Devi displayed a prodigious talent for mathematics. Her father C V Sundararaja Rao worked as a trapeze artist, lion tamer, tight rope walker and magician in a circus. He discovered his daughter's ability to memorize numbers while teaching her a card trick when she was about three years old. Her father left the circus and took her on road shows that displayed her ability at calculation. She did this without any formal education. At the age of 6, she demonstrated her arithmetic abilities at the University of Mysore. She was the first women mathematician in India. In 1944, Devi moved to London, UK. She travelled to several countries around the world demonstrating her arithmetic talents. She was on tour to Europe throughout 1950. Devi returned to India in mid 1960's and married with Paritosh Bannergi, an officer of Indian Administrative Service in Kolkata. Devi's life changed after marrying a Gay man and finally divorced in 1979. She had only one daughter namely Anupama Bannergi who married to Ajay Abhaya Kumar with whom she has two daughters and lives in London. In 1980 Devi, contested the Lok Sabha Elections as an independent Candidate for Mumbai South and for Medak in AP, (now in Telangana) in Medak. She stood against the former Prime Minister Indira Gandhi saying she wanted to defend the people of Medak from being fooled by Mrs. Gandhi. She came 9^{th} with 6,514 votes (1.47% of votes). Devi returned to Banglore in early 1980's. In 1988 she travelled to US to have her abilities studied by "Arthur Jensen" a professor of educational psychology at the University of California, Berkely. Jensen tested her performance at several tasks including the calculation of large numbers. Jensen published his findings in the academic journal *Intelligence* in 1990. Shakuntala Devi's life inspired many individuals particularly women to pursue their interests in mathematics and overcome societal barriers.

7.2 Contributions

Shakuntala Devi made significant contribution in the field of mathematics, particularly in the area of mental calculations and number theory. Her remarkable talent was her ability to perform Complex calculations mentally and with great speed. She could solve calculations involving large numbers and intricate operations including multiplication, division, square roots and logarithms faster than conventional methods or even computers. For extraordinary mental arithmetic skills demonstrated the potential of human computation and garnered worldwide attention. In 1977 at Southern Methodist University, she gave the 23rd root of a 201-digit number in 50 seconds. She answered 546,372,891 which was was confirmed by calculations done at the US Bureau of standards by the Universal Automatic Computer 1101, computer for which a special program had to be written to perform such a large calculation, which took longer time than her to do the same. On 18th June 1980, she demonstrated the multiplication of two 13-digit numbers 7,686,369,744,870 into 2,465,099,745,779. These numbers were picked at random by the department of computing at Imperial college London. She correctly answered 18,947,668,177,995,426,462,773,730(26 digits) in 28 seconds. This event was recorded in 1982 Guinness Book of World Records. Writer Steven Smith commented, "the result is so far to anything previously reported that it can only be described as unbelievable". At Stanford University US, 1988 she calculated cube root of 95,443,993 as 457 in 2 seconds and cube root of 2,373,927,704 as 1334 in 10 seconds. Also she calculated the 8th root of 200,471,612,231,936 as 46 in 10 seconds. As per "New York times", an American daily newspaper, founded and continuously published in New York City "She could give you the cube root of 188,132,517 or almost any other number in the time it took to ask the question". If you gave her any date in the last century, she would tell you what day of the week it fell on. She was praised as the authentic heroine of her times and she could command the headlines in newspapers and magazines. Guinness World Records honoured Indian math genius Shakuntala Devi with the long overdue record title for ''fastest human computation'', four decades after she achieved the feat, the certification was received by Anupama Banerji, daughter of the late mathematician. The fastest human computation recorded is 28 seconds which was achieved by Shakuntala Devi when she multiplied two randomly selected 13-digit numbers, at Imperial College London, UK, on June 18, 1980. Devi's incredible mental arithmetic earned her the title of "Human Computer". This title was given to her when she appeared in an interview with BBC channel. This show was hosted by Leslie Mitchell on Oct. 5 1950. The channel had asked a very difficult question to her to which she replied correctly but the channel was not having the same answer and hence they described the answer incorrect. However, when they checked it later, the answer was absolutely correct as given by Shakuntala Devi. Thereby she got the title as human computer and become house hold name but she never liked the title human computer. As a female mathematician, Shakuntala Devi broke barriers and shattered stereotypes in a male dominated field. Her achievement and successes inspired countless women to pursue their interests in mathematics and challenge societal norms. She became a symbol of empowerment for women in STEM (Science, Technology, Engineering and Mathematics) and demonstrated that gender should never be a barrier to pursuing ones passion. Despite not having formal mathematical training Shakuntala Devi offered several books on mathematics. Her books such as *"Figuring :The joy of numbers"*, Shakuntala Devi dramatizes the endless fascination of numbers and their ability to amaze and entertain. She offers easy-to-learn short cuts on how to add long columns in your head, multiply, divide and find square roots quickly, almost magically. Fractions, decimals and compound interest become clear and easy to deal with. The author takes delight in working out huge problems mentally and sometimes even faster than computers. In *Figuring* she shares her secrets with you. Another famous book which she wrote was *"Mathability: Awaken the math genius in your child"*, attempts to demystify the many myths surrounding mathematics, in order to invoke the reader's interest on this highly misunderstood subject. The author first addresses the mixed reactions and idiosyncrasies that people have developed over the years. The book comprises of many captivating pictures to stir up interest in the minds of young readers. It also contains many mythological stories and real-life examples that shed light upon the importance of mathematics and why it is interesting, when approached with the right spirit and motivation. She simplified complex mathematical ideas and provided practical examples to promote a deeper understanding and appreciation of mathematics. Her works also include, *More puzzles to puzzle, Astrology for you, Books of numbers, In the wonderland of numbers, Perfect murder and super memory: it can be used*. Shakuntala Devi was an advocate for mathematical education and believed in the importance of cultivating mathematical skills. She emphasised that mathematics was not just a subject but a valuable tool for logical thinking, problem solving and understanding the world around us. Through her performance, lectures and books, she encouraged individuals, particularly children, to develop their mathematical abilities and overcome their fear of the subject. She was found to be an expert in highly complex mental arithmetic. Her passion to expand the human capacity made her develop the concept known as 'Mind Dynamics'. In 1977 she wrote the book, *The World of Homosexuals*. The first published academic study of homosexuality in India, for which she was criticised. In this book, she wrote: "Marriage is a contract between two individuals who commit themselves to each other to enjoy life and to see both their children and their loved ones through thick and thin, for the rest of their lives, whether they like it or not". In the documentary *For Straights Only,* she said that her interest in the topic was because of her marriage to a homosexual man and her desire to look at homosexuality more closely to understand it. The book, considered 'pioneering', features interviews with two young Indian homosexual men, a male couple in Canada seeking legal marriage, a temple priest who explains his views on homosexuality and a review of the existing literature on homosexuality. It ends with a call for decriminalisation of homosexuality and 'full and complete acceptance-not tolerance and sympathy'. The book however went unnoticed at that time. She started writing short stories and murder mysteries, and had a keen interest in music. Shakuntala Devi, a great mathematician also started astrology. By studying stars, planets of an individual as per the date of birth, she used to make predictions about personal lives, describe their personalities, and offered advice. She has also written a book on astrology- *Astrology for You.* The book gives information on planets, zodiacs etc.

Devi won the 'Distinguished Women of year award' in 1969 from the University of Philippines along with the Gold medal. In 1988 she was honoured with the 'Ramanujan Mathematical Genius Award' in Washington DC conferred to her by the then Indian Ambassador to US. The genius had setup Shakuntala Devi Education Foundation Public Trust with the aim of providing quality education to children of deprived sections of the society. The trust also runs a college in HSR Layout in Banglore. In addition, a new Mathematics Research Block has been constructed at a cost of Rs. 5 crore by the trust. Talking about her dream of opening a mathematics University she had said 'It is my dream to open a mathematical University and R&D centre, which will educate a cross section of people, using modern techniques, short cuts and smart methods. I cannot transfer my abilities to anyone but I can think of quicker ways with which to help people develop numerical aptitude. There is large number of people whose logic is unexplored'. A month before her death, she was honoured with the 'Lifetime Achievement Award' in Mumbai in 2013. The Lifetime Achievement Award recognizes individuals or groups who have made significant and sustained contributions to the development and/or dissemination of the ethical principles that govern research by way of their scholarship, administration, leadership, or mentoring. Shakuntala Devi was extremely talented in calculations and her achievements /books show her contribution to maths. Her ability to calculate accurately with enormous speed has never been seen before. She used to beat the speed of computers in calculations.

In April 2013 this great mathematician and astrologist was admitted in hospital in Bangalore with severe respiratory problems. Over the following two weeks she had heart and kidney complications. She died in hospital on 21th April 2013 at the age of 83. Her last rites were conducted in presence of hundreds of people including relatives and admirers at Banshoankari Crematorium on Sunday evening. On 4^{th} Nov. 2013, Devi was honoured with 'Google Doogle' on what has been her 84^{th} birthday. A Google Doodle is a special, temporary alteration of the logo on Google's homepages intended to commemorate holidays, events, achievements, and notable historical figures. A film on her life titled Shakuntala Devi was announced on May 2019.The film stars Vidya Balan in the lead title role and featured Sanya Malhotra, Amit Sadh and Jisshu Singh Gupta in the supporting roles, produced by Sony Pictures Network Production. The film streamed worldwide on Amazon Prime Video on 31th July 2020.

Maryam Mirzakhani (1977-2017)

Chapter 8

Maryam Mirzakhani

8.1 Life History

Maryam Mirzakhani was an Iranian mathematician who was the first women to receive the Fields Medal, which is one of the highest honors in mathematics. The Fields Medal is regarded as something akin to a Nobel Prize for mathematics. It was established by Canadian mathematician John Fields and comes with a 15,000 Canadian dollar (£8,000) cash prize.

Maryam Mirzakhani was born on 12 May 1977 in Tehran, Iran. She displayed an early aptitude for mathematics and developed a passion for the subject during the childhood.

Maryam Mirzakhani attended Farzanegan School, a highly regarded middle and high school for gifted girls, in Tehran, where she excelled in mathematics.

In 1994 she earned a gold medal at the International Mathematical Olympiad (IMO) in Hong Kong, becoming the first female Iranian student to achieve this distinction. The following year in Toronto, she became the first Iranian student to achieve the full score and win two gold medals in the International Mathematical Olympiad. Later in her life, she collaborated with friend, colleague and Olympiad silver medalist, Roya Beheshti Zavareh on their book "*Elementary Number Theory, Challenging Problems*" (in Persian) which was published in 1999.

Roya Beheshti Zavareh, an Associate Professor of Mathematics at Washington University, Maryam's childhood friend, played an important role in her life journey.

Mirzakhani and Zavareh together were the first women to compete in the Iranian National Mathematical Olympiad and won gold and silver medals respectively.

After completing high school, Maryam Mirzakhani entered Sharif University of Technology in Tehran, one of the Iran's top Universities, where she studied Mathematics. She graduated in 1999 with a Bachelor's degree in Mathematics. During her time there, she received recognition from the American Mathematical Society for her work in developing a simple proof of the theorem of Schur.

Maryam Mirzakhani then pursued her graduate studies abroad, attending Harvard University in the United States.

At Harvard, Maryam Mirzakhani worked under the guidance of the renowned Mathematician Curtis T.McMullen.

In 2004, she earned her doctorate from Harvard University for her 130-page thesis titled "Simple geodesics on hyperbolic Riemann surfaces and volume of the moduli space of curves." Geodesics are the natural generalization of the idea of a "Straight line" to "Curved spaces". She was awarded the Leonard M and Eleanor B Blumenthal award for her thesis, which was judged as outstanding

Prof. Dame Frances Kirwan, a member of the medal selection committee from the University of Oxford, pointed out that despite mathematics being viewed traditionally as "a male preserve", women have contributed to mathematics for centuries.

She noted that around 40% of mathematics undergraduates in the UK are women, but that proportion declines rapidly at PhD level and beyond. Prof. Kirwan said:

"I hope that this award will inspire lots more girls

and young women, in this country and around the

world, to believe in their own abilities and aim to

be the Fields Medalists of the future,"

Prof. Sir John Ball, another British mathematician and a former president of the IMU, agreed that Prof. Mirzakhani's win was "fantastically important". Speaking to BBC News from the congress in Seoul, South Korea, he said that a female winner was overdue and that Prof. Mirzakhani is one of many brilliant women mathematicians.

He added that the committee had an unenviable job choosing the winners. "These four are really deserving of this recognition, but of course any work at this level also builds on exceptional work by other people."

Prof. Mirzakhani's seminal research concerns shapes called Riemann surfaces. These are convoluted mathematical objects that can be analyzed using complex numbers – i.e. numbers with real and imaginary parts.

In particular, she has studied "moduli spaces" of these shapes, which map all of the possible geometries of a Riemann surface into their own, new space.

In 2008 Maryam Mirzakhani married Jan Vondrak, a Czech theoretical computer scientist and applied mathematician who currently is a professor at Stanford University. They had a daughter; Mirzakhani lived in Palo Alto, California.

Mirzakhani described herself as a 'slow' mathematician, saying that you have to spend some energy and effort to see the beauty of mathematics.

To solve problems Mirzakhani would draw doodles on sheets of paper and write Mathematical formulas around the drawings. Her daughter described her mother's work as painting.

She declared:

I don't have any particular recipe (for developing new proofs). It is like being lost in a jungle and trying to use all the knowledge, that you can gather to come up with some new tricks and with some luck, you might find a way out.

Overall, Maryam Mirzakhani's early life and education laid the foundation for her remarkable career as a mathematician and her ground breaking contributions to the field, have left a lasting impact on the world of mathematics.

Maryam Mirzakhani made significant contributions to the field of Mathematics, particularly in the area of hyperbolic geometry and complex dynamics.

Her work had a profound impact on the study of moduli spaces, Teichmuller theory and the understanding of the geometry of Riemann surfaces.

8.2 Contributions

Here are some of the notable contributions of Maryam Mirzakhani.

Research on Technical Theory

Maryam Mirzakhani made groundbreaking contributions to the understanding of Teichmuller theory, a branch of mathematics that deals with the geometric properties of Riemann surfaces. She developed novel techniques and approaches to study the multi space of Riemann surfaces, which has applications in various areas of mathematics including theoretical physics.

Hyperbolic Geometry

Mirzakhani's research focused on the geometry of hyperbolic surfaces. She explored the behavior of geodesics (curve that locally minimize length) on hyperbolic surfaces, providing insights into the intricate and complex nature of their behavior. Her work led to breakthroughs in our understanding of the moduli space of hyperbolic surfaces.

Dynamics of Mapping space

Mirzakhani made significant contributions to the study of the dynamics of mapping spaces, specifically the behavior of billiard paths within polygons. She developed a deep understanding of the trajectories of billiard balls on various types of surfaces including flat surfaces and hyperbolic surfaces. Her work connected different area of mathematics, such as dynamics geometry and mathematical physics.

Maryam Mirzakhani became the first women to be awarded the prestigious Fields Medal, often referred to as the Nobel Prize of mathematics. The award recognized her groundbreaking contributions to the understanding of the geometry and dynamics of Riemann Surfaces. Mirzakhani's achievement not only highlighted her exceptional mathematical abilities but also served as an inspiration for aspiring mathematician particularly women around the world.

Maryam Mirzakhani was a strong advocate for increasing the participation of women in mathematics, she actively encouraged and supported women pursuing careers in mathematics, particularly in her home country of Iran.

Mirzakhani's achievements and visibilities as a successful mathematician helped challenge gender stereotypes in academia and inspired countless young women to pursue their passion for mathematics.

Maryam Mirzakhani's work extended into the field of topology where she made significant contributions to the study of the topology of multi spaces. She explored the connectivity properties of these spaces and the relationship between their topology and the geometry of Riemann surfaces.

Maryam Mirzakhani's research also included contribution to the field of Ergodic theory, which studies the statistical properties of dynamical systems. She explored the ergodic properties of billiard paths and other dynamical systems, providing new insights into the statistical behavior of these systems.

Maryam Mirzakhani passed away on July 14, 2017 at the age of 40, after a battle with breast cancer. Her death was a great loss to the mathematical community and she was mourned by mathematicians and scientists around the world. In recognition of her contributions to the field of mathematics, The International Mathematical Union established the Maryam Mirzakhani prize in 2019 which is awarded every four years to exceptional young female mathematicians and students around the world and her contributions to the field of mathematics will continue to be recognized and celebrated for many years to come.

The Mathematics community lost one of the brightest stars, a woman who could inspire many people, particularly all the girls, to follow their dreams to succeed. It is worthwhile to mention that the International Council for Science declared Maryam Mirzakhani's birthday, 12 May, as International day for Women in Mathematics in respect of her memory.

In 2022, the University of Oxford launched the Maryam Mirzakhani Scholarships, which provide support for female mathematicians pursuing doctoral studies at the university.

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