

Paper II: Recent Advances in Applied Mathematics

Max. Marks: 100 (25 marks for each unit)

Time: 3 Hours

Course Outcomes

After completing this course, students will be able to:

CO1: Apply transform techniques and modern numerical methods to solve ordinary and partial differential equations.

CO2: Use R software for data management, statistical analysis, visualization, and implementation of computational algorithms.

CO3: Explain the principles of machine learning and implement basic models for regression, classification, dimensionality reduction, and probabilistic inference.

CO4: Develop and analyze mathematical models for biological and biomedical systems involving heat transfer, mass transport, and physiological processes.

Unit I: Bio-Mathematical Modelling

Concepts and significance of mathematical and computational modelling in biology and medicine. Principles of heat and mass transfer in biological systems, including diffusion, convection, perfusion, and bioheat transfer. The Stewart–Hamilton method for the measurement of cardiac output. Mathematical modelling of mass transport in the human body, with emphasis on oxygen and carbon dioxide exchange across biological membranes.

Unit II: Mathematical Techniques

Review of Laplace Transform and Fourier series with applications to ordinary and partial differential equations. Numerical methods for solving ordinary and partial differential equations. Solution techniques for partial differential equations using the Explicit Finite Difference Method, Crank–Nicolson Method, Finite Element Method (FEM).

Unit III: R Programming for Scientific Computing

Introduction to the R programming language. R as a scientific calculator; vectorization, notation, and naming conventions. Creation and manipulation of vectors, lists, factors, and data frames. Extraction operators, data import/export, data manipulation, descriptive statistics, and graphical visualization. Matrix operations in R, matrix attributes, subsetting techniques, and implementation of computational programs using R.

Unit IV: Fundamentals of Machine Learning

Introduction to machine learning and its applications. The learning problem and paradigms of learning: supervised, unsupervised, and reinforcement learning. Learning feasibility, error analysis, and noise. Linear models for regression and classification. Bayesian inference and probabilistic learning. Principal Component Analysis (PCA).

Suggested Books/Papers:

1. K. W. Morton and D. F. Mayers, Numerical Solution of Partial Differential Equations: An Introduction, Cambridge University Press.
2. Norman Matloff, The Art of R Programming, No Starch Press.
3. Learning from Data Abu-Mostafa, Y.S., Malik Magdon-Ismael & Lin H.-T. (2012) New York, NY: AMLBOOK.
4. Ethem Alpaydin, Introduction to Machine Learning, MIT Press.
5. James D. Murray, Mathematical Biology: I. An Introduction, Springer.

Paper III: Research Topics in Modelling Biomechanics

Max. Marks: 100 (25 marks for each unit)

Time: 3 Hours

Course Objectives

The course aims to enable research scholars to:

- CO1:** Develop mathematical models of physiological transport phenomena.
- CO2:** Apply analytical and computational techniques to biomechanical systems.
- CO3:** Understand fractional calculus and its applications in biological modelling.
- CO4:** Study cardiovascular mechanics and biofluid dynamics using classical and modern approaches.

Unit I: Heat and Mass Transfer Modelling in Biological Tissues

Introduction to biomechanics and biological transport phenomena. Conservation laws and governing equations, Mathematical models for oxygen, nutrient, and drug transport, Initial and boundary value problems in biological systems, Analytical and numerical solution techniques.

Unit II: Fractional-Order Differentiation in Biomechanical Models

Fundamentals of fractional calculus and memory effect, Fractional integral and derivative operators. Caputo, Riemann–Liouville. Fractional ordinary and partial differential equations. Fractional bioheat and diffusion models. Existence & Stability of fractional models. Numerical methods for fractional differential equations.

Unit III: Cardiovascular Modelling and Biofluid Mechanics

Fundamentals of biofluid mechanics. Continuity and Navier–Stokes equations. Laminar and pulsatile blood flow. Poiseuille's law: derivation, assumptions, and applications. Mathematical models of arterial and microvascular flow. Fluid–structure interaction in blood vessels. Applications to cardiovascular diseases and biomedical devices.

Unit IV: Recent Advances in Biomechanical Modelling

1. Enso Ndreko et al.(2025), Modeling of bio-heat transfers in lungs with fractional models, Annual Reviews in Control 60, 101010.
2. Ahsan ul Haq & MA Khanday, (2022), Mathematical analysis of oxygen and carbon dioxide exchange in the human capillary and tissue system, Computer Methods in Biomechanics and Biomedical Engineering, 26(2), 199-208, <https://doi.org/10.1080/10255842.2022.2053115>

Recommended Books/Papers

1. Podlubny, I. (1999). Fractional Differential Equations: An Introduction to Fractional Derivatives, Fractional Differential Equations, Methods of Their Solution and Some of Their Applications. Academic Press, San Diego, California, USA. ISBN: 978-0125588409.
2. Quarteroni, A., Formaggia, L., and Veneziani, A. (Editors) (2009). Cardiovascular Mathematics: Modeling and Simulation of the Circulatory System. Springer-Verlag, Milan, Italy. ISBN: 978-8847011502.
3. Aref, M. H. F., Hussein, A. A., and El-Sharkawy, Y. H. (2025). Fractional-Order Bioheat Modeling for Enhanced Prediction of Tissue Response in Thermal Therapies. *Journal of Thermal Biology*, Vol. 133, Elsevier, Article 104287.
4. Afzal, U., Shah, N. A., Zeeshan, and Chung, J. D. (2024). Thermal Memory and Moving Linear Thermal Shocks on Heat Transfer within Biological Tissues: An Atangana–Baleanu Fractional Integral. *Journal of Thermal Analysis and Calorimetry*, Vol. 149, Springer Nature.
5. Tasnim, Z. J., and Nasrin, R. (2024). *Thermal Wave and Pennes' Models of Bioheat Transfer in Human Skin: A Transient Comparative Analysis*. Heliyon, Vol. 10, No. 21, Elsevier, Article e39756.

PhD Course Work Syllabus-2026 (Gazala Salam/Insha Rasool)

Paper III: Research Topics in Modeling Population Growth and Decay

Max. Marks: 100 (25 marks for each unit)

Time: 3 Hours

Course Objectives

After successful completion of this course, students will be able to:

- develop mathematical understanding of logistic, Gompertz, delay, Lotka–Volterra, competition, mutualism, and host–parasite models, with emphasis on stability.
- provide knowledge of age-structured population dynamics using Leslie matrix models, including population projection, stable age distribution, eigenvalue analysis, and applications in conservation biology, fisheries, forestry, and demography.
- familiarize students with fuzzy mathematical modelling techniques for population dynamics, including fuzzy sets, membership functions, fuzzy inference systems, and applications of uncertainty modelling in population growth and decay processes.

Unit I: Single-Species Population Models

Logistic and Gompertz growth model, Stability and bifurcation analysis. Lotka–Volterra predator-prey model, Local and global stability.

Introduction to age-structured population models, Construction of the Leslie matrix, Stable age distribution, Harvesting and management strategies in age-structured populations, Applications to wildlife conservation, fisheries, forestry, and human demography.

Unit II: Probability Convergence and Asymptotic Theory

Modes of convergence of random variables, including convergence in probability, almost sure convergence, convergence in distribution, and convergence of moments. Weak Law of Large Numbers (WLLN) and its conditions, Strong Law of Large Numbers (SLLN) with examples, and the applications of Central Limit Theorem.

Unit III: Theory and Methods of Statistical Estimation

Basic concepts and applications in statistical inference. Point and interval estimation, and desirable properties of estimators including unbiasedness, consistency, efficiency, and sufficiency. Cramér–Rao Lower Bound (CRLB), Minimum Variance Unbiased (MVU) estimation, and complete and minimal sufficient statistics. Methods of estimation: Method of Moments, Maximum Likelihood Estimation (MLE), Minimum Chi-Square Estimation.

Unit IV: Fundamentals of Fuzzy Sets and Applications to Population Modelling

Introduction to fuzzy and classical (crisp) sets. Membership functions (Triangular, Trapezoidal, and Gaussian), basic set operations, α -cuts, fuzzy relations. Fuzzification, rule aggregation, and defuzzification methods (Centroid and Bisector). Applications to population growth and decay models, including fuzzy exponential and logistic models, uncertainty in birth, death, and carrying capacity parameters, and fuzzy parameter estimation in ecological systems.

Recommended Books/Papers

1. Mathematical Models in Population Biology and Epidemiology, Springer.
2. Pelagalli, C., Faccio, S., & Casari, P. (2025). Mathematical Modeling and Simulation of Logistic Growth. *Applied Sciences*, 15(8), 4409.
3. Yuan, Y., Xu, H., & Lin, Z. (2025). Extinction and Persistence in a Logistic Model with Birth and Harvesting Impulses. *Journal of Applied Analysis and Computation*.
4. Rosenbaum, B., et al. (2023). Confronting Population Models with Experimental Microcosm Data: From Trajectory Matching to State-Space Models. *Ecosphere*.
5. He, X., Ni, W.-M., & Wang, H. (2023). Dynamics of Consumer–Resource Reaction–Diffusion Models: Single and Multiple Consumer Species. *Journal of Mathematical Biology*.

Paper III: Research Topics in Modelling Neuroscience

Max. Marks: 100 (25 marks for each unit)

Time: 3 Hours

Course Objective

Upon successful completion of this course, students will be able to:

- Develop a strong understanding of the mathematical principles underlying neuronal dynamics, membrane electrophysiology, and neural signaling.
- Study and analyze classical and modern mathematical models of neurons, action potential generation, and synaptic transmission.
- Implement and simulate neuronal models using Python for computational analysis, visualization, and interpretation of neuronal behavior.
- Apply computational modelling techniques to investigate normal neuronal function and neurological disorders arising from ion channel dysfunction.

Unit I: Mathematical Foundations of Neuronal Dynamics

Introduction to neuroscience and the importance of mathematical modelling in understanding neuronal function. Review of diffusion processes and Fick's laws of diffusion. Mathematical description of the neuron and its equivalent electrical circuit. Membrane potential and ionic equilibrium. Resting membrane potential. Nernst equation and Goldman–Hodgkin–Katz (GHK) voltage equation. RC, Integrate-and-Fire neuron model. Hodgkin–Huxley model: formulation, gating variables, ion channel dynamics, and modelling of action potential generation.

Unit II: Advanced Neuronal and Synaptic Models

Fitz Hugh–Nagumo model: mathematical formulation, phase-plane analysis, and simplification of Hodgkin–Huxley dynamics. Cable equation and electrical signal propagation in dendrites and axons. Multi-compartment neuron models and spatial modelling of neurons. Introduction to chemical synapses and mechanisms of synaptic transmission. Excitatory and inhibitory synapses. Neurotransmitters, receptors, and postsynaptic potentials. Modelling of synaptic receptor kinetics using the Magleby–Stevens model.

Unit III: Computational Modelling Using Python

Python programming for scientific computing using NumPy, SciPy, Matplotlib, Pandas, and Jupyter Notebook. Numerical solution of neuronal differential equations using Euler and Runge–Kutta methods. Implementation and visualisation of Integrate-and-Fire and Hodgkin–Huxley neuron models. Parameter sensitivity and bifurcation analysis of neuronal excitability. Simulation of ion channel mutations associated with neurological disorders. Introduction to the Brian2 simulator for spiking neural networks.

Unit IV: Some latest papers in Neuroscience

1. Pang, J. C. S., & Bantang, J. Y. (2015). Hodgkin–Huxley neurons with defective and blocked ion channels. *International Journal of Modern Physics C*, 26(10), 1550112.
2. Gandolfi, D., Boiani, G. M., Bigiani, A., & Mapelli, J. (2021). Modeling neurotransmission: Computational tools to investigate neurological disorders. *International Journal of Molecular Sciences*, 22(9), 4565.

Recommended Books/Papers

1. Keener, J., & Sneyd, J. (Eds.). (2009). *Mathematical physiology: II: Systems physiology*. New York, NY: Springer New York.
2. Ermentrout, B., & Terman, D. M. (2010). *Mathematical foundations of neuroscience* (Vol. 35). New York: Springer.
3. Guttag, J. V. (2016). *Introduction to computation and programming using Python: With application to understanding data*. MIT press.
4. Abu-Mostafa, Y. S., Magdon-Ismael, M., & Lin, H. T. (2012). *Learning from data* (Vol. 4, p. 4). New York: AMLBook.
5. Govindaraju, D., Subbian, S., & Narayanan, S. N. (2025). Computational modelling for risk assessment of neurological disorder in diabetes using Hodgkin-Huxley model. *Computer methods and programs in biomedicine*, 267, 108799.