

2-Year/1-Year Master's Program in Mathematics (NEP-2020)

RIEMANNIAN GEOMETRY

Master's Mathematics (4th Semester)

Course Code: MMTHDRG426 (15 hours per credit)

Continuous Assessment: Marks 30, Theory: Marks 70

Total Credits: **04**

Total Marks: **100**

Time Duration: 2½ hrs

Course Learning Outcomes(CLO's): Upon successful completion of this course, students will be able to:

CLO1. Understand and explain the fundamental concepts of differentiable manifolds, tangent spaces, and smooth maps.

CLO2. Analyze Riemannian metrics and compute geometric quantities such as connections, covariant derivatives, and geodesics..

CLO3. Apply the concepts of curvature, including sectional curvature and the Riemann curvature tensor, to study the geometric properties of Riemannian manifolds.

CLO4. Examine submanifolds of Riemannian manifolds and evaluate geometric structures such as the second fundamental form and mean curvature.

Unit I

Topological manifolds, Differentiable atlas, Smooth maps, Diffeomorphism, Equivalent atlases, Differentiable structure on a manifold, Space of smooth maps, Tangent vectors and tangent space, Differential of a smooth map.

Unit II

Immersion, Embedding and Submanifolds, Vector fields, Lie algebra of vector fields, Integral curve of a vector field, Covectors and Cotangent spaces, Pull back of a linear differential form, Covariant and Contravariant tensors, Laws of transformation for the components of tensors.

Unit III

Differential forms, Exterior product, Grassman algebra of forms, Exterior derivative, Affine Connection, Parallelism, Geodesic Covariant differentiation of tensors, Torsion and Curvature of a Connection, Structure equation of Cartan, Bianchi's identities.

Unit IV

Partition of unity, Riemannian metric and Riemannian manifolds, Riemannian or Levi-Civita connection, Riemannian curvature, Ricci and Scalar curvature.

CLO-PLO Mapping Matrix (Strength version)

CLO 	PLO 	PLO1	PLO2	PLO3	PLO4	PLO5	PLO6	PLO7	PLO8	Average CLO
MMTHDRG426.1		3	3	2	2	3	2	1	3	2.37
MMTHDRG426.2		3	3	2	2	3	2	2	3	2.5
MMTHDRG426.3		3	3	3	1	3	2	2	3	2.5
MMTHDRG426.4		3	3	3	3	3	2	2	3	2.75
Average PLO		3	3	2.5	2	3	2	1.75	3	2.53

Recommended books:

1. J. M. Lee, Introduction to smooth manifolds, Graduate Texts in Mathematics, Springer.
2. B.-Y Chen, Geometry of Submanifolds, Dover Publications, 1973
3. W. M. Boothby, An introduction to Differential Manifolds and Riemannian Geometry, Academic Press; 2nd edition, 1986.
4. S. Kumaresan: A Course in Differential Geometry and Lie groups, Hindustan Book Agency.
5. M. P. do-Carmo, Riemannian geometry, Birkhäuser, Boston, Basel, Berlin, 1992