2-Year Master's Program in Mathematics (NEP-2020)
TOPICS IN NUMBER THEORY

Master's Mathematics (1st Semester) Total Credits: 04
Course Code: MMTHDNT126 (15 hours per credit) Total Marks: 100
Continuous Assessment: Marks 30, Theory: Marks 70 Time Duration: 2½ hrs

Course Learning Outcomes(CLO's): After the completion of this course, students shall be able to:
CLO1. Apply concepts of modular arithmetic, primitive roots, and the λ-function to solve classical number theoretic problems.
CLO2. Analyze and interpret quadratic residues using Euler's criterion, Legendre and Jacobi symbols, and quadratic reciprocity.
CLO3. Evaluate number theoretic functions, use Möbius inversion and investigate perfect numbers and rational approximations.
CLO4. Utilize continued fractions and L-functions to explore connections between rational approximations and prime distribution.

UNIT -I
Integers belonging to a given exponent (mod p) and related results, converse of Fermat's theorem; If $d|p-1$, then congruence $x^d \equiv 1(mod\ p)$ has exactly d-solutions; If any integer belongs to $t(mod\ p)$, then exactly $\phi(t)$ incongruent numbers belong to $t(mod\ p)$, primitive roots, there are $\phi(p-1)$ primitive roots of an odd prime p, any power of an odd prime has a primitive root, the function $\lambda(m)$ and its properties, $a^{\lambda(m)} \equiv 1(mod\ m)$, where $(a, m) = 1$. There is always an integer which belongs to $\lambda(m)(mod\ m)$, primitive λ -roots of m, the numbers having primitive roots are $1, 2, 4, p^\alpha, 2p^\alpha$ where p is an odd prime.

UNIT -II
Quadratic residues, Euler criterion, the Legendre symbol and its properties, Lemma of Gauss, the law of a quadratic reciprocity, characterization of primes of which 2,-2, 3,-3, 5, 6 and 10 are quadratic residues or non-residues, Jacobi symbol and its properties, the reciprocity law for Jacobi symbol.

UNIT -III
Number theoretic functions, some simple properties of $\tau(n), \sigma(n), \phi(n)$ and $\mu(n)$. Mobius inversion formula. Perfect numbers. Necessary and sufficient condition for an even number to be perfect, the function $[x]$ and its properties, average order of magnitudes of $\tau(n), \sigma(n), \phi(n)$, Farey fractions, rational approximations.

UNIT -IV
Simple continued fractions, application of the theory of infinite continued fractions to the approximation of irrationals by rationals, Hurwitz theorem, Relation between Riemann Zeta function and the set of primes, characters, the L-Function $L(S, \chi)$ and its properties, Dirichlet's theorem on infinity of primes in an arithmetic progression.

CLO-PLO Mapping Matrix (Strength version)

CLO	PLO	PLO1	PLO2	PLO3	PLO4	PLO5	PLO6	PLO7	PLO8	Average CLOs
MMTHDNT126.1		3	3	2	3	2	3	3	3	**2.75**
MMTHDNT126.2		3	2	3	2	2	2	2	3	**2.37**
MMTHDNT126.3		3	1	3	1	1	2	2	3	**2**
MMTHDNT126.4		3	2	2	2	3	2	1	3	**2.25**
Average PLOs		**3**	**2**	**2.5**	**2**	**2**	**2.25**	**2**	**3**	**2.34**

Recommended Books

1. Ivan Niven, Herbert S. Zuckerman, and Hugh L. Montgomery, An Introduction to the Theory of Numbers, John Wiley & Sons, 6th edition, 1991.
2. W. J. LeVeque, Topics in Number Theory, Vol. I & II, Addison-Wesley 1956.
3. T. M. Apostol, Introduction to Analytic Number Theory, Springer International, 1976.
4. G.H. Hardy and E.M. Wright, An Introduction to the Theory of Numbers, Oxford University Press, 6th edition, 2008.