2-Year/1-Year Master's Program in Mathematics (NEP-2020)

NON-LINEAR FUNCTIONAL ANALYSIS

Master's Mathematics (4th Semester) Total Credits: **04**
Course Code: MMTHDNL426 (15 hours per credit) Total Marks: **100**
Continuous Assessment: Marks 30, Theory: Marks 70 Time Duration: 2½ hrs

Course Learning Outcomes(CLO's): Upon successful completion of this course, students will be able to:

CLO1. Analyze the properties of convex sets and functions, non-expansive and monotone operators, and apply key fixed point theorems such as Browder's and Minty's Theorem.

CLO2. Understand and apply concepts of differentiability in Banach spaces, including Gateaux and Fréchet derivatives, subdifferentials, and their role in convex analysis.

CLO3. Explore the geometry of Banach spaces through uniform and strict convexity, smoothness, and duality maps, and establish connections with reflexivity and inequalities.

CLO4. Apply variational principles, minimization techniques, and iterative methods such as Mann and Ishikawa to solve variational inequalities and optimization problems using results like the Lax-Milgram Lemma and Lions-Stampacchia Theorem.

Unit-I

Convex sets, convex functions, lower semicontinuous convex functions, non-expansive operators, characterization of projection onto convex sets and their geometrical interpretation, fixed points of non-expansive operators, Browder fixed point theorem, Monotone operators, maximal monotone operator and their properties, resolvents of monotone operators, Minty theorem,.

Unit-II

Gateaux Derivative, Frechet Derivative, subdifferential of convex functions, directional derivatives, characterization of convexity and strict convexity, directional derivatives and subgradients, Gateaux and Frechet differentiability, differentiability and continuity.

Unit-III

Uniformly convex spaces, strictly convex Banach spaces, the modulus of convexity, uniform convexity, strict convexity and reflexivity, inequalities in uniformly convex spaces, smooth spaces, the modulus of smoothness, uniformly smooth spaces, inequalities in uniformly smooth, duality maps in Banach spaces, characterization of some real Banach spaces by the duality map.

Unit-IV

Billinear forms and its applications, Lax-Milgram lemma, minimization of functionals, variational inequalities, relationship between abstract minimization problems and variational inequalities, Lions Stampacchia theorem for existence of solution of variational inequality, Ekeland's variational principle and its applications to fixed point theorems and optimization, Takahashi's minimization theorem, Mann and Ishikawa iterative methods.

CLO-PLO Mapping Matrix (Strength version)

CLO ⇩ PLO ⇨	PLO1	PLO2	PLO3	PLO4	PLO5	PLO6	PLO7	PLO8	Average CLO
MMTHDNL426.1	3	3	3	1	3	2	2	3	**2.5**
MMTHDNL426.2	3	3	3	2	3	2	2	3	**2.63**
MMTHDNL426.3	3	3	3	2	3	2	3	3	**2.75**
MMTHDNL426.4	3	3	3	3	3	3	3	3	**3**
Average PLO	**3**	**3**	**3**	**2**	**3**	**2.25**	**2.5**	**3**	**2.72**

Recommended Books:

1. 1. H. H. Bauschke and P. L. Combettes, Convex Analysis and Monotone Operator Theory in Hilbert Spaces, Springer New York, 2011.
2. D. Kinderlehrer and G. Stampacchia, An Introduction to Variational Inequalities and Their Applications, Academic Press, New York, 1980.
3. A.H. Siddiqi, K. Ahmed and P. Manchanda, Introduction to Functional Analysis with Applications, Anamaya Publishers, New Delhi-2006.
4. M. C. Joshi and R. K. Bose, Nonlinear Functional Analysis and its Applications, WilleyEastern Limited, 1985.
5. C.E. Chidume: Geometric Properties of Banach Spaces and Nonlinear Iterations, Springer-
6. Verlag, London, 2009.
7. V. Berinde: Iterative Approximation of Fixed Points, Springer-Verlag, Berlin Heidelberg, 2007.