2-Year/1-Year Master's Program in Mathematics (NEP-2020)

ADVANCED MEASURE THEORY

Master's Mathematics (4^{th} Semester) Total Credits: **04**
Course Code: MMTHDMT426 (15 hours per credit) Total Marks: **100**
Continuous Assessment: Marks 30, Theory: Marks 70 Time Duration: 2½ hrs

Course Learning Outcomes(CLO's): Upon successful completion of this course, students will be able to:

CLO1. Understand and differentiate between semi-rings, rings, algebras, and σ-algebras of sets, and construct measures and outer measures on various set structures.

CLO2. Analyze finite and σ-finite measure spaces, extend measures uniquely to σ-algebras, and apply the Monotone Class Theorem in the construction of product measures.

CLO3. Relate improper Riemann integrals to Lebesgue integrals, and compute integrals using approximation techniques and the Riemann–Lebesgue Lemma.

CLO4. Characterize absolutely continuous functions in terms of Lebesgue integrals, apply the fundamental theorem of calculus in the context of Lebesgue integration, and understand the structure and inequalities in Lp spaces.

UNIT -I

Semi-ring, ring, algebra and σ- algebra of sets, measures on semi-rings, examples of various measure spaces, outer measure associated with a set function and basic properties, measurable sets associated with an outer measure as a σ- algebra, outer measure induced by a measure.

UNIT -II

Finite and σ- finite measure spaces and relationship, measurable sets of finite measure space, extension of measure to σ- algebra and its uniqueness, non-measurable sets. Product measures and product σ- algebra, measurable rectangles, monotone class and elementary sets, Monotone class theorem.

UNIT -III

Improper Riemann integral, criteria for improper Riemann integral as a Lebesgue integral, calculation of some improper Riemann integrable functions viz: $\int_0^\infty e^{-x^2}dx$; $\int_0^\infty e^{-x^2}\cos(2xt)dx,\ \ t \in R$, $\int_0^\infty \frac{\sin(x)}{x}e^{-xt}dx, t \geq 0$; $\int_0^\infty \frac{e^{-xt}}{x}dxt > 0$. Approximation of integrable functions, Riemann-Lebesgue lemma.

UNIT -IV

For $f \in L_1[a,b], F' = f, a.e.$ on $[a,b]$. If f is absolutely continuous on [a, b] with $f' = 0$ a.e., then f is constant. Characterization of an absolutely continuous function as an indefinite Lebesgue integral. Non-Lebesgue integrability of f where $f(x) = x^2\sin(1/x^2), f(0) = 0$ on [0,1]. Fundamental theorem of calculus for the Lebesgue integral. A brief introduction to Lp spaces. Holder's and Minkowki's inequalities.

CLO-PLO Mapping Matrix (Strength version)

CLO ⇩ PLO ⇨	PLO1	PLO 2	PLO3	PLO4	PLO 5	PLO6	PLO7	PLO8	Average CLO
MMTHDMT426.1	3	3	3	3	3	2	1	3	**2.62**
MMTHDMT426.2	3	3	3	2	3	3	2	3	**2.75**
MMTHDMT426.3	3	2	3	2	3	2	2	3	**2.5**
MMTHDMT426.4	3	2	3	2	3	2	3	3	**2.63**
Average PLO	**3**	**2.5**	**3**	**2.25**	**3**	**2.25**	**2**	**3**	**2.63**

Recommended Books:

1. C. D. Aliprantis and O. Burkinshaw, Principles of Real Analysis, Academic Press Inc. 2nd Edition (1990).
2. Chae, S.B., Lebesgue Integration, Springer Verlag, 2nd Edition (1995).
3. Rudin, W., Principles of Mathematical Analysis, McGraw Hill (2023).
4. Barra, De. G., Measure theory and Integration, New Age International Publishers, 3rd Edition (2022).
5. Rana , I. K., An Introduction to Measure and Integration, Narosa Publications 2nd Edition (2005).