2-Year/1-Year Master's Program in Mathematics (NEP-2020)

MEASURE THEORY

Master's Mathematics (3rd Semester) Total Credits: 04
Course Code: MMTHDMT326 (15 hours per credit) Total Marks: 100
Continuous Assessment: **Marks 30,** Theory: **Marks 70** Time Duration: **2½ hrs**

Course Learning Outcomes(CLO's): Upon successful completion of the course, the students will be able to:

CLO1. Understand and apply the fundamental concepts of measure theory, including outer measure, Borel sets, Lebesgue measurability, and the existence of non-measurable sets.

CLO2. Characterize measurable functions, and analyze convergence properties such as almost everywhere convergence, convergence in measure, and Egoroff's theorem.

CLO3. Develop a strong foundation in Lebesgue integration, including the equivalence between Riemann and Lebesgue integrals, and apply important theorems like Fatou's lemma and the Dominated Convergence Theorem.

CLO4. Study the properties of absolute continuity, bounded variation, and their relationships, and apply Vitali's covering lemma to solve problems involving differentiation of monotone functions.

UNIT -I

Measure theory: definition of outer measure and its basic properties, outer measure of an interval as its length, countable additivity of the outer measure, Borel measurable sets and Lebesgue measurability of Borel sets, Cantor set, existence of non- measurable sets and of measurable sets which are not Borel, outer measure of monotonic sequences of sets.

UNIT -II

Measurable functions and their characterization, algebra of measurable functions, Stienhauss theorem on sets of positive measure, Ostroviski's theorem on measurable solution of $f(x+y)=f(x)+f(y), x, y \in R$, convergence a.e., convergence in measure and almost uniform convergence, their relationship on sets of finite measure, Egoroff's theorem.

UNIT -III

Lebesgue integral of a bounded function, equivalence of $L-$integrability and measurability for bounded functions, Riemann integral as a Lebesgue integral, basic properties of Lebesgue – integral of a bounded function, fundamental theorem of calculus for bounded derivatives, necessary and sufficient condition for Riemann integrability on $[a,b]$, $L-$integral of non-negative measurable functions and their basic properties, Fatou's lemma and monotone convergence theorem, $L-$integral of an arbitrary measurable function and basic properties, dominated convergence theorem and its applications.

UNIT -IV

Absolute continuity and bounded variation, their relationships and counter examples, indefinite integral of an L-integrable function and its absolute continuity, necessary and sufficient condition for bounded variation, Vitali's covering lemma and a. e., differentiability of a monotone function f and $\int f' \leq f(b)-f(a)$.

CLO-PLO Mapping Matrix (Strength version)

CLO ⇩ PLO ⇨	PLO1	PLO2	PLO3	PLO4	PLO5	PLO6	PLO7	PLO8	Average CLO
MMTHCMT326.1	3	3	2	2	3	1	2	3	**2.38**
MMTHCMT326.2	3	3	2	2	2	1	2	3	**2.25**
MMTHCMT326.3	3	3	3	2	3	3	3	3	**2.88**
MMTHCMT326.4	3	3	3	3	2	3	1	3	**2.63**
Average PLO	**3**	**3**	**2.5**	**2.25**	**2.5**	**2**	**2**	**3**	**2.54**

Recommended Books:

1. P.K. Jain, V.P. Gupta, Lebesgue measure and Integration, John Wiley and Sons, 1986.
2. R. Goldberg, Methods of Real Analysis**,** Dover Publications, 2nd Edition, 1980.
3. G. De Barra, Measure Theory and Integration**,** Narosa Publishing House,1st Edition, 2003.
4. W. Rudin, Principles of Mathematical Analysis, McGraw-Hill Education, 3rd Edition, 1976.
5. Chae, Lebesgue Integration, Springer, 1st Edition, 2000.
6. S. M. Shah and Saxena, Real Analysis, Discovery Publishing House, 1st Edition,1994.