2-Year Master's Program in Mathematics (NEP-2020)
LINEAR ALGEBRA

Masters Mathematics (2nd Semester) Total Credits: 04
Course Code: MMTHDLA226 (15 hours per credit) Total Marks: 100
Continuous Assessment: Marks 30, Theory: Marks 70 Time Duration: 2½ hrs

Course Learning Outcomes(CLO's):After the completion of this course, students are expected to

CLO1. Analyze linear operators using eigenvalues, eigenvectors, invariant subspaces, and polynomial techniques to determine diagonalization and structural properties.

CLO2. Apply concepts of inner product spaces, including orthogonality, projections, and Gram–Schmidt process, to study least squares problems and special classes of operators.

CLO3. Examine the structure of linear operators through generalized eigenvectors, Jordan canonical form, and decompositions such as SVD for classification up to similarity.

CLO4. Analyze bilinear and quadratic forms, their matrix representations, and transformations under change of basis, including structure-preserving properties.

Unit – I

Similarity of matrices. Invariant subspaces and restriction of operators. Eigenvalues and eigenvectors. Eigenspaces and diagonalization of operators. Polynomials applied to operators: minimal polynomial and characteristic polynomial, and their role in describing operator behavior. Cayley– Hamilton theorem. Multiplicity theory: algebraic and geometric multiplicities and their role in diagonalization. Existence of upper-triangular matrix representations. Nilpotent operators and their basic structure.

Unit – II

Inner product spaces: norms, orthogonality, and orthonormal bases. Gram–Schmidt orthogonalization. Orthogonal complements and orthogonal projections. Best approximation and least squares problems. The adjoint operator T^* and its algebraic properties. Self-adjoint (Hermitian), normal, and unitary operators. Spectral theorem for normal operators and diagonalization via orthonormal bases. Positive operators and their properties. The Riesz Representation Theorem.

Unit – III

Generalized eigenvectors and generalized eigenspaces. Nilpotent operators and the index of nilpotency. Primary decomposition of vector spaces. Jordan canonical form: existence, uniqueness, and structure. Rational canonical form (brief introduction). Classification of linear operators up to similarity. Polar decomposition and Singular Value Decomposition.

Unit – IV

Bilinear forms on vector spaces: definition, examples, and matrix representation with respect to a basis. Symmetric and skew-symmetric bilinear forms and their basic properties. Change of basis and equivalence of bilinear forms. Quadratic forms associated with symmetric bilinear forms and their elementary properties. Groups preserving bilinear forms, including orthogonal groups and their role in preserving structure.

CLO-PLO Mapping Matrix (Strength version)

CLO ⬇ PLO ➡	PLO1	PLO2	PLO3	PLO 4	PLO5	PLO6	PLO7	PLO8	Average CLO
MMTHCLA226.1	3	3	2	3	3	3	2	3	**2.75**
MMTHCLA226.2	3	2	2	2	3	3	2	3	**2.5**
MMTHCLA226.3	3	2	2	2	3	3	2	3	**2.5**
MMTHCLA226.4	3	2	2	2	3	3	3	3	**2.63**
Average PLO	**3**	**2.25**	**2**	**2.25**	**3**	**3**	**2.25**	**3**	**2.59**

Recommended Books:

1. Sheldon Axler, Linear Algebra Done Right, Springer, 4th Edition.
2. Stephen H. Friedberg, Arnold J. Insel, Lawrence E. Spence, Linear Algebra, Pearson, 5th Edition
3. Kenneth Hoffman, Ray Kunze, Linear Algebra, Prentice Hall, 2nd Edition.
4. Serge Lang, Linear Algebra, Springer, 2nd Edition.
5. Carl D. Meyer, Matrix Analysis and Applied Linear Algebra, SIAM, 2nd Edition.