2-Year Master's Program in Mathematics (NEP-2020)

INTEGRAL EQUATIONS & CALCULUS OF VARIATIONS

Master's Mathematics (1st Semester) Total Credits: 04
Course Code: MMTHDIE126 (15 hours per credit) Total Marks: 100
Continuous Assessment: Marks 30, Theory: Marks 70 Time Duration: 2½ hrs

Course Learning Outcomes(CLO's):Upon successful completion of this course, the student will be able to:

CLO1. Understand and solve linear integral equations of Volterra and Fredholm types using methods like successive approximations and Neumann series.

CLO2. Apply Fredholm theory, symmetric kernel techniques, and Hilbert-Schmidt theory to analyze and solve integral equations.

CLO3. Formulate and solve variational problems using Euler-Lagrange equations and related necessary and sufficient conditions.

CLO4. Employ direct methods such as Rayleigh-Ritz and Galerkin's to solve variational problems in ordinary and partial differential equations.

UNIT -I

Linear integral equations of the first and second kinds, Volterra and Fredholm integral equations, relations between differential and integral equations, solution of Volterra and Fredholm integral equations by the methods of successive substitutions and successive approximations, iterated and resolvent kernels, Neumann series, reciprocal functions, Volterra's solutions of Fredholm equations.

UNIT -II

Fredholm associated equation, solution of integral equations using Fredholm's determinant and minor, homogeneous integral equations, integral equations with separable kernels, the Fredholm alternatives, symmetric kernels, Hilbert Schmidt theory for symmetric kernels, applications of integral equations to differential equations, initial value problem, boundary value problem, Dirac-Delta function, Green's function approach.

UNIT -III

Functionals, The concept of variation and its properties, Variational problems with fixed and moving boundaries, The Euler-Lagrange equation, Variational problems in parametric form. Reflection and refraction extremals. Necessary and Sufficient conditions for an extremum, Canonical equations and variational principles, Complementary variational principles.

UNIT -IV

Isoperimetric problem, The Hamilton-Jacobi equation, Direct methods for variational problem: Rayleigh-Ritz method, Galerkin's method, Variational methods for boundary value problems in ordinary and partial differential equations.

CLO-PLO Mapping Matrix (Strength version)

CLO ↓ PLO →	PLO1	PLO2	PLO3	PLO4	PLO5	PLO6	PLO7	PLO8	Average CLO
MMTHDIE226.1	3	3	3	2	2	3	2	2	**2.5**
MMTHDIE226.2	3	3	1	3	2	1	3	3	**2.38**
MMTHDIE226.3	3	3	2	1	2	3	2	3	**2.38**
MMTHDIE226.4	3	3	2	2	2	2	3	3	**2.5**
Average PLO	**3**	**3**	**2**	**2**	**2**	**2.25**	**2.5**	**2.75**	**2.44**

Books Recommended:

1. R. P. Kanwal, Linear Integral Equations (Theory and Technique), Academic Press Birkhauser-1997.
2. W. V. Lovitt, Linear Integral Equations, Dover Publications, Inc. New York, 1950.
3. K. F. Riley, M.P. Hobson and S.T. Bence, Mathematical Methods for Physics and Engineering Cambridge University Press, U.K., 1997.
4. M. D. Raisinghania, Integral Equations and Boundary Value Problems, S.C. Chand India, 2007.
5. Shanti Swarup, Integral Equations (&Boundary Value Problems), Krishna Prakashan Media (P) Ltd. Meerut, India, 2014.
6. A. S. Gupta, Calculus of Variation with Applications, Prentice-Hall, India, 1997.
7. G. M. Ewing, Calculus of Variations with Applications, Dover, 1985.
8. Sagan, An Introduction to the Calculus of Variations, Dover Publications, 1967.
9. J.N. Reddy, Finite Element Method, McGraw Hills, Third Edition, 2005.