

2-Year/1-Year Master's Program in Mathematics (NEP-2020)

HOMOLOGICAL ALGEBRA

Master's Mathematics (4th Semester)

Total Credits: **04**

Course Code: MMTHDHA426 (15 hours per credit)

Total Marks: **100**

Continuous Assessment: Marks 30, Theory: Marks 7

Time Duration: 2½ hrs

Course Learning Outcomes(CLO's): After the completion of this course, students will be able to:

CLO1. Demonstrate a deep understanding of ring theory concepts including ideals, homomorphisms, radicals, and apply results like the Chinese Remainder Theorem and Prime Avoidance Lemma in algebraic structures.

CLO2. Analyze modules over rings, explore their structures using tools such as exact sequences, tensor products, localization, and comprehend important results like Nakayama's Lemma.

CLO3. Understand chain conditions on rings and modules, apply the Hilbert Basis Theorem, and explore primary decomposition in Noetherian and Artinian rings.

CLO4. Apply the language of category theory including functors, natural transformations, and exactness in abelian categories to study algebraic structures in a more general setting.

Unit-I

Review of modules, sub-modules, module homomorphisms. Finitely generated modules, Nakayama lemma, exact sequences, free and projective modules, Tensor product of modules, flat modules. Restriction and extension of scalars. Rings and modules of fractions and localization.

Unit-II

Chain conditions, Noetherian rings. Hilbert basis theorem, primary decomposition in Noetherian rings. Artinian rings and related results, modules over PID. Modules of finite length and primary Decomposition.

Unit-III

Definitions and examples of categories, functors and natural transformations, adjoint functors, definition of an additive category, kernels and cokernels, abelian categories, exact sequences, exact, left exact, and right exact functors.

Unit-IV

Projective and injective objects, the category of complexes, snake lemma, cohomology functors. Projective and injective resolutions, examples, null-homotopies, uniqueness of resolutions up to homotopy, definition and basic properties of derived functors, Tohoku viewpoint (universal delta-functors).

CLO-PLO Mapping Matrix (Strength version)

CLO  PLO 	PLO1	PLO2	PLO3	PLO4	PLO5	PLO6	PLO7	PLO8	Average CLO
MMTHDHA426.1	3	3	3	2	3	1	2	3	2.5
MMTHDHA426.2	3	3	3	3	3	2	1	3	2.63
MMTHDHA426.3	3	3	3	2	3	2	3	3	2.75
MMTHDHA426.4	3	3	3	3	3	2	3	3	2.88
Average PLO	3	3	3	2.5	3	1.75	2.25	3	2.69

Recommended Books

1. Charles A. Weibel, An introduction to homological algebra, Cambridge Studies in Advanced Mathematics, Edition 1, 1995.
2. M. F. Atiyah and I. G. Macdonald, Introduction to Commutative Algebra, Addison-Wesley, 1969.
3. E. Kunz, Introduction to Commutative Algebra and Algebraic Geometry, Birkhäuser, 2015.