2-Year/1-year Master's Program in Mathematics (NEP-2020)
ADVANCED GRAPH THEORY

Master's Mathematics (3rd Semester) Total Credits: **04**
Course Code: MMTHDGT326 (15 hours per credit) Total Marks: **100**
Continuous Assessment: Marks 30, Theory: Marks 70 Time Duration: 2½ hrs

Course Learning Outcomes(CLO's):

CLO1: Understand and apply graph coloring concepts (vertex, edge, map) and related theorems (Brook's, Vizing's) to analyze graph structures and solve real-world problems.
CLO2: Understand types, properties, and applications of graph matchings and apply key theorems (Hall's, Tutte's, f-factor) to solve optimization and structural problems.
CLO3: Analyze edge and total graphs, eccentricity sequences, and apply structural theorems (Whitney's, Lesniak's) to study graph properties and existence problems.
CLO4: Explore connections between graphs and algebra (groups, matrices, spectra) using theorems (Frucht's) and linear algebra tools to study graph structure and behavior.

UNIT- I

Colorings: Vertex coloring, chromatic number ($\boldsymbol{G}$), bounds for $\boldsymbol{\chi}(\boldsymbol{G})$, Brooks theorem, edge coloring, Vizing's theorem, map coloring, six color theorem, five color theorem, every graph is four colourable iff every cubic bridgeless plane map is 4-colorable, every planar graph is 4-colorable iff $\boldsymbol{\chi}'(\boldsymbol{G}) = \mathbf{3}$. Heawood map coloring theorem, uniquely colorable graphs.

UNIT – II

Matchings: Matchings and 1-factors, Berge's theorem, Hall's theorem, 1-factor theorem of Tutte, antifactor sets, construction of g', f-factor theorem (statement only), f-factor theorem implies 1-factor theorem, Erdos- Gallai theorem follows from f-factor theorem, degree factors, **k**-factor theorem, factorization of $\mathbf{K_n}$.

UNIT - III

Edge graphs and eccentricity sequences: Edge graphs, Whitney's theorem on edge graphs, Beineke's theorem, edge graphs of trees, edge graphs and traversibility, total graphs, eccentricity sequences and sets, Lesniak theorem for trees, construction of trees, neighbourhoods, Lesniak theorem graphs.

UNIT -IV

Groups in graphs and graph spectra: Automorphism groups of graphs, graph with a given group, Frucht's theorem, Cayley digraph, spectrum of a graph, spectrum of some graphs- complete graph, complete bipartite graph, path, cycle, determinant of adjacency matrix, Sach's theorem, regular graph and its compliment, edge graph. Laplacian spectrum and Laplacian energy (definition and examples only).

CLO-PLO Mapping Matrix (Strength version)

CLO ⇩ PLO ⇨	PLO1	PLO2	PLO3	PLO4	PLO5	PLO6	PLO7	PLO8	Average CLO
MMTHDGT326.1	3	3	2	2	3	3	2	3	**2.63**
MMTHDGT326.2	3	3	3	2	3	3	2	3	**2.75**
MMTHDGT326.3	3	3	3	3	3	2	3	3	**2.88**
MMTHDGT326.4	3	3	3	3	3	3	1	3	**2.75**
Average PLO	**3**	**3**	**2.75**	**2.5**	**3**	**2.75**	**2**	**3**	**2.75**

Recommended Books:

1. R. Balakrishnan, K. Ranganathan, A Text Book of Graph Theory, Springer-Verlag, New York (2012).
2. B. Bollobas, Extremal Graph Theory,Springer (2002).
3. F. Harary, Graph Theory, Narosa (2001).
4. Narsingh Deo, Graph Theory with Applications to Eng. and Comp. Sci, PHI. (1979)
5. S. Pirzada, An Introduction to Graph Theory, Universities Press, Orient Blackswan, (2012).
6. W. T. Tutte, Graph Theory, Cambridge University Press. (2016).
7. D. B. West, Introduction to Graph Theory, Pearson (2022).