

2-Year/1-Year Master's Program in Mathematics (NEP-2020)

FOURIER AND WAVELET ANALYSIS

Master's Mathematics (3rd Semester)

Total Credits: **04**

Course Code: MMTHDFW326 (15 hours per credit)

Total Marks: **100**

Continuous Assessment: Marks 30, Theory: Marks 70

Time Duration: 2½ hrs.

Course Learning Outcomes(CLO's): Upon completion of this course, students will be able to

CLO1. Understand the fundamentals of Fourier transforms and apply them to solve differential equations.

CLO2. Analyze and apply continuous and discrete wavelet transforms, including their basic properties and key examples.

CLO3. Explore multi resolution analysis (MRA) and construct orthonormal wavelet bases using scaling functions.

CLO4. Examine advanced wavelet structures such as spline, Daubechies and biorthogonal wavelets, and their applications.

Unit I

Fourier Transforms and Their Applications: Definition and examples of Fourier transforms in $L^2(\mathbf{R})$, basic properties of Fourier transforms, Convolution theorem, Plancherel and Parseval's formulae, Poisson summation formula, Shannon-Whittaker sampling theorem, Heisenberg's uncertainty principle, Applications of Fourier transforms to ordinary and partial differential equations (Laplace, heat and wave equations).

Unit II

Wavelet Transforms: Definition and examples of windowed Fourier transform, Continuous wavelet transform and its basic properties, wavelets by convolution, Haar wavelet, Morlet wavelet, Mexican hat wavelet, resolution of the wavelet transform, Parsevals' and inversion formulae for wavelet transform, characterization of range, Discrete wavelet transform with examples.

Unit III

Multiresolution Analysis: Multiresolution analysis (MRA), properties of scaling functions, construction of orthonormal wavelet bases, examples of MRA based wavelets (Haar, Shannon and Meyer wavelets), fast wavelet transform, applications of basic equations, characterization of MRA wavelets and scaling functions.

Unit IV

Elongations of MRA-Based Wavelets: Construction and examples of Spline wavelets (Franklin and Battle-Lemarié wavelets), compactly supported wavelets, Daubechies wavelets, biorthogonal wavelets, wavelet packets, Harmonic wavelets.

CLO-PLO Mapping Matrix (Strength version)

CLO ↓ PLO	PLO1	PLO2	PLO3	PLO4	PLO5	PLO6	PLO7	PLO8	Average CLO
MMTHDFW326.1	3	2	2	2	3	3	2	3	2.5
MMTHDFW326.2	3	3	2	1	3	3	2	3	2.5
MMTHDFW326.3	3	3	3	2	3	3	2	3	2.75
MMTHDFW326.4	3	2	3	3	3	3	1	3	2.63
Average PLO	3	2.5	2.5	2	3	3	1.75	3	2.6

Recommended Books:

1. Jonas Gomes and Luiz Velho, From Fourier Analysis to Wavelets, Springer, 2015.
2. M. Pinsky, Introduction to Fourier Analysis and Wavelets, Brooks/Cole Publishing, 2002.
3. F.A. Shah and A.Y. Tantary, Wavelet Transforms: Kith and Kin, CRC Press, 2023.
4. A. Henandez and G. Weiss, A First Course on Wavelets, CRC Press, 1996.
5. L. Debnath and F. A. Shah, Wavelet Transforms and Their Applications, Springer, 2015.
6. I. Daubechies , Ten Lectures on Wavelets, SIAM, Philadelphia, 1992.