2-Year/1-Year Master's Program in Mathematics (NEP-2020)

FIELDS AND GALOIS THEORY

Master's Mathematics (4th Semester) Total Credits: **04**
Course Code: MMTHCFG426 (15 hours per credit) Total Marks: **100**
Continuous Assessment: Marks 30, Theory: Marks 70 Time Duration: 2½ hrs

Course Learning Outcomes(CLO's): Upon successful completion of this course, students will be able to:

CLO1. Develop an understanding of prime fields, field extensions, and algebraic elements by applying concepts like the remainder and factor theorems, Kronecker's theorem, and algebraically closed fields in the study of polynomial roots and algebraic integers.

CLO2. Construct and analyze splitting fields of polynomials, distinguish between separable and inseparable extensions, and apply results such as the primitive element theorem and the theory of perfect fields to classify extensions.

CLO3. Investigate the structure of finite fields using Galois groups and Moore's theorem, analyze the multiplicative group of a finite field, and study automorphisms of field extensions and their fixed fields in the context of Galois theory.

CLO4. Apply advanced results of Galois theory including the fundamental theorem, normal and cyclic extensions, and solvability by radicals, and explore classical problems of field theory such as constructions with straightedge and compass.

Unit I

Review of fields and their characteristics, Prime fields and their structure, field extensions, finite, algebraic and simple field extensions. Algebraic elements and algebraic integers, roots of polynomials, Remainder and Factor theorems, Kronecker's theorem and applications.

Unit II

Splitting field of a polynomial, existence and uniqueness of splitting field of a polynomial, separable and inseparable polynomials, separable and inseparable extensions, simplicity of finite separable extensions, primitive element theorem, Perfect fields and related results.

Unit III

Automorphisms of field extensions, Galois groups, Galois fields and examples, Moore's theorem on finite fields, structure of the multiplicative group of a finite field, fixed fields, order of fixed field.

Unit IV

Algebraically closed fields and related results, Normal extensions, Necessary and sufficient conditions for finite separable extension to be normal, Galois extensions and examples, Fundamental theorem of Galois theory, Constructions with straight edge and compass.

CLO-PLO Mapping Matrix (Strength version)

CLO ↓ PLO →	PLO1	PLO2	PLO3	PLO4	PLO5	PLO6	PLO7	PLO8	Average CLO
MMTHCFG426.1	3	3	2	2	3	2	1	3	**2.38**
MMTHCFG426.2	3	3	2	1	3	1	3	3	**2.38**
MMTHCFG426.3	3	3	3	2	3	3	2	3	**2.75**
MMTHCFG426.4	3	3	3	3	3	2	1	3	**2.63**
Average PLO	**3**	**3**	**2.5**	**2**	**3**	**2**	**1.75**	**3**	**2.54**

Recommended Textbooks

1. D. S. Dummit and R. M. Foote, Abstract Algebra – Wiley, 3rd Edition, 2004.
2. I. N. Herstein,Topics in Algebra, Wiley India, 2nd Edition, 2006.
3. Ian Stewart, Galois Theory, Chapman and Hall/CRC, 4th Edition, 2015.
4. D. J. H. Garling, A Course in Galois Theory – Cambridge University Press, 2013