2-Year/1-Year Master's Program in Mathematics (NEP-2020)
FUNCTIONAL ANALYSIS

Master's Mathematics (3rd Semester) Total Credits: **04**
Course Code: MMTHCFA326 (15 hours per credit) Total Marks: **100**
Continuous Assessment: Marks 30, Theory: Marks 70 Time Duration: 2½ hrs

Course Learning Outcomes(CLO's): Upon completion of this course, students will be able to

CLO1. Distinguish between Banach and Hilbert spaces, and analyze their structural and topological differences.

CLO2. Apply the concept of orthogonal decomposition in Hilbert spaces and examine the role of orthonormal sets and sequences in functional spaces.

CLO3. Represent bounded linear functionals using inner products and interpret the Riesz Representation Theorem in Hilbert spaces.

CLO4. Develop a foundational understanding of linear operators and functional spaces, with an emphasis on their applications in mathematical and scientific problem-solving.

UNIT-I

Banach Spaces: Normed space, definition and examples. Banach spaces, subspaces, quotient spaces, incompleteness of $C[a, b]$ under the integral norm, spaces of bounded linear operators and completeness, bounded linear functionals, finite dimensional Banach spaces, equivalent norms, equivalence of norms on finite dimensional normed spaces, dual of a normed linear space, duals of $c_0, l_p(p \geq 1), C[0,1]$.

UNIT-II

Fundamental Theorems: Hahn Banach theorem (extension form) and its applications, Uniform boundedness and weak boundedness, Banach-Steinhauss theorem, open mapping and closed graph theorems, counterexamples to Banach-Steinhauss, open mapping theorem and closed graph theorems for incomplete domain and range spaces, conjugate of a continuous linear operator and its properties, separable Banach spaces and the separability of some concrete Banach spaces $c_0, l_p(p \geq 1), C[0,1]$, reflexive Banach Spaces, closed subspace and the dual of a reflexive Banach space, examples of reflexive and non-reflexive Banach spaces, weak convergence.

UNIT-III

Hilbert spaces: definition and examples, Cauchy's Schwartz inequality, parallogram law, orthonormal systems, Bessel's inequality and Parseval's Identity for complete orthonormal systems, Gram Schmidt process, orthonormal basis in separable Hilbert spaces, projection onto convex sets.

UNIT-IV

Functionals and Operators on Hilbert Spaces**:** Projection theorem, Riesz Representation theorem, Hilbert property of the dual of a Hilbert space and counter examples of incomplete inner product spaces, reflexivity of Hilbert space, Hilbert adjoint operator (existence, uniqueness and properties), weak convergence and Bolzano-Weirstrass property in Hilbert Spaces, self-adjoint, normal and unitary operators and their properties, finite dimensional spectral theorem for normal operators.

CLO-PLO Mapping Matrix (Strength version)

CLO ⇩ PLO ⇨	PLO1	PLO 2	PLO3	PLO4	PLO5	PLO6	PLO7	PLO8	Average CLO
MMTHCFA326.1	3	3	3	2	3	2	1	3	**2.5**
MMTHCFA326.2	3	3	3	1	2	3	3	3	**2.63**
MMTHCFA326.3	3	3	3	2	2	3	2	3	**2.63**
MMTHCFA326.4	3	2	2	3	2	2	3	3	**2.5**
Average PLO	**3**	**2.75**	**2.75**	**2.25**	**2.25**	**2.5**	**2.25**	**3**	**2.57**

Recommended Books:

1. Erwin Kreyszig, Introductory Functional Analysis with Applications, Wiley Publishers, reprint 2015.
2. B. V. Limaya, Functional Analysis, New Age International Pvt. Ltd; 3rd edition, 2014.
3. C. Goffman & G. Pedrick, A First Course in Functional Analysis, American Mathematical Society; 2nd edition, 2017.
4. J.B. Conway, A Course in Functional Analysis, Springer, 4th Edition, 1994.