2-Year/1-Year Master's Program in Mathematics (NEP-2020)

DIFFERENTIAL GEOMETRY

Master's Mathematics (3rd Semester)	Total Credits: 04
Course Code: MMTHCDG326 (15 hours per credit)	Total Marks: 100
Continuous Assessment: Marks 30, Theory: Marks 70	**Time Duration: 2½ hrs**

Course Learning Outcomes(CLO's): Upon successful completion of the course, the students will be able to:

CLO1. Analyze and characterize curves in terms of their curvature, torsion, and Frenet-Serret frame, including applications to space curves and helices.

CLO2. Understand and compute key concepts of regular surfaces, including the first and second fundamental forms, and analyze the geometric properties of surfaces like normal curvature and principal curvatures.

CLO3. Apply differential geometry concepts to study Gaussian and mean curvature, minimal surfaces, isometries, and coordinate systems.

CLO4. Derive and apply the Gauss-Bonnet theorem, geodesic equations, and fundamental theorems for surfaces, and solve geodesic problems on different surfaces.

UNIT-I

Parametrized Curves, Regular curves, Arc length, Reparameterization by arc-length, Arc-length is independent of parameterization, plane and space curves, Curvature, Frenet-Serret frame. Characterization of plane and space curves in terms of their curvature and torsion. Fundamental theorem of space curves. Helix, necessary and sufficient condition for a space curve to be helix. Involutes and evolutes of space curves. Global properties of curves: Simple closed curves, Isoperimetric Inequality, Four Vertex Theorem.

UNIT -II

Regular surfaces, Change of coordinates. The Tangent Plane, The First Fundamental Form (metric), Area element, and properties of metric, Orientable and Non-Orientable surfaces. Gauss map, differential of the Gauss map is self-adjoint operator. Second Fundamental form. Normal curvature, Normal curvature in terms of second fundamental form. Principal curvatures and principle directions, Euler's formula. Gaussian and mean curvature, Classification of points with prescribed principal curvatures.

UNIT-III

Gaussian curvature in terms of area, Hilbert's theorem and its applications in terms of Liebmann theorem. Weingarten equation, lines of curvature, Rodrigue's formula for line of curvature. Joachimsthal's Theorem. Ruled and Minimal surfaces, Isothermal coordinates. A isothermal surface is minimal if and only if its coordinate functions are harmonic. Minimal surfaces and Holomorphic functions: Weirstrass-Enneper Representation. Isometry between surfaces, local isometry, and characterization of local isometry.

UNIT -IV

Christoffel symbols, Theorema Egregium. Gauss equations and Mainardi Codazzi equations for surfaces, fundamental theorem for regular surface. (Statement only). Vector Fields, Geodesics: geodesic curvature, geodesic curvature is intrinsic, equations of geodesic, geodesic on sphere and pseudo sphere, geodesic as distance minimizing curves. Gauss-Bonnet theorem (statement only), geodesic triangle on sphere, implication of Gauss-Bonnet theorem for geodesic triangle.

CLO-PLO Mapping Matrix (Strength version)

CLO ⇓ **PLO** ⇒	PLO1	PLO2	PLO3	PLO4	PLO5	PLO6	PLO7	PLO8	**Average CLO**
MMTHCDG326.1	3	3	2	1	3	2	3	3	**2.5**
MMTHCDG326.2	3	3	2	2	3	2	1	3	**2.38**
MMTHCDG326.3	3	3	3	1	3	2	1	3	**2.38**
MMTHCDG326.4	3	3	3	3	3	2	3	3	**2.88**
Average PLO	**3**	**3**	**2.5**	**1.75**	**3**	**2**	**2**	**3**	**2.54**

Recommended Books:

1. M. P. Do Carmo, Differential geometry of curves and surfaces, Dover Publications, 2nd Ed. 2016.
2. Sebastian Montiel, Antonio Ros, Curves and surfaces, AMS, 2nd Edition, 2009.
3. K.K. Dube, Differential geometry and Tensors, I.K. International Publishing House, 2009.
4. Andrew Pressley, Elementary Differential Geometry, Springer Undergraduate Mathematics Series, 2nd edition , 2010.
5. Kristopher Tapp, Differential geometry of curves and surfaces, Springer, 2016.
6. J. McCleary, Geometry from a Differentiable Viewpoint, Cambridge, 2nd Edition.