2-Year/1-Year Master's Program in Mathematics (NEP-2020)
ADVANCED COMPLEX ANALYSIS

Master's Mathematics (3^{rd} Semester) Total Credits: 04
Course Code: MMTHCCA326 (15 hours per credit) Total Marks: 100
Continuous Assessment: Marks 30, Theory: Marks 70 Time Duration: 2½ hrs

Course Learning Outcomes (CLO's): After successful completion of this course, students will be able to:

CLO1. Apply advanced integration techniques in the complex plane, including Cauchy's residue theorem, contour integrals, Jordan's lemma, Parseval's identity, and related results, to evaluate real and complex integrals.

CLO2. Analyze the convergence of power series using Cauchy–Hadamard formula, understand Picard's theorem, and explore the principles and uniqueness of analytic continuation, Hadamard's three-circle theorem, reflection principles, and boundary behaviors of analytic functions.

CLO3. Employ Poisson integral formulas for the circle and half-plane, apply Poisson–Jensen formula, study uniqueness theorems, and investigate results such as Borel and Carathéodory theorem, Bloch's and Schottky's theorems, and Picard's little theorem.

CLO4. Explore functions with positive real part, univalent functions, Borel's theorem, the area theorem, Bieberbach's conjecture, convex hull and Gauss–Lucas theorem, and applications including critical points and the Gresgorin Disk theorem.

UNIT -I

Calculus of residues, Cauchy's residue theorem, evaluation of integrals by the method of residues, Jordan's inequality, Jordan's lemma and applications, Mittag-Leffler's expansion theorem, Parseval's identity, Blaschke's theorem.

UNIT -II

Power series: Cauchy–Hadamard formula for the radius of convergence, a power series represents an analytic function within the circle of convergence, Picard's theorem on power series: If $a_n > a_{n+1} \geq 0$ and $\lim a_n = 0$, then the series $\Sigma\, a_n z^n$ has radius of convergence equal to 1 and the series converges for $|z| = 1$ except possibly at $z = 1$. Hadamard–Pringsheim theorem, the principle of analytic continuation, uniqueness of analytic continuation, power series method of analytic continuation, functions with natural boundaries e.g. $\Sigma zn!$, Σz^{2n}, Schwarz reflection principle.

UNIT -III

Poisson integral formula for circle and half-plane, Poisson–Jensen formula, Estermann's uniqueness theorem, Hadamard's three circle theorem, $\log M(r)$ and $\log I_2(r)$ as convex functions of $\log r$, Theorem of Borel and Carathéodory, Bloch's and Schottky's theorems (statements only), Picard's little theorem.

UNIT -IV

Functions with positive real part, Borel's theorem, univalent functions, area theorem, Bieberbach's conjecture (statement only) and Koebe's ¼ theorem, critical points in terms of zeros, convex hull and Gauss–Lucas theorem, applications of Gauss–Lucas theorem, Gresgorin disk theorem.

CLO-PLO Mapping Matrix (Strength version)

CLO ⇩ / PLO ⇨	PLO1	PLO2	PLO3	PLO4	PLO5	PLO6	PLO7	PLO8	Average CLO
MMTHDCA326.1	3	3	3	2	3	3	1	3	**2.63**
MMTHDCA326.2	3	3	2	1	3	1	2	2	**2.13**
MMTHDCA326.3	3	3	3	3	2	3	2	2	**2.63**
MMTHDCA326.4	3	3	3	3	3	1	3	3	**2.75**
Average PLO	**3**	**3**	**2.75**	**2.25**	**2.75**	**2**	**2**	**2.5**	**2.53**

Recommended Books:

1. Ahlfors, Complex Analysis, McGraw Hill (2000).
2. E. C. Titchmarsh, Theory of Functions, Oxford University Press, 2nd Edition (1976).
3. J. B. Conway, Functions of a Complex Variable-1, Springer, 2nd Edition (1995).
4. Richard Silverman, Complex Analysis, Dover Publications Inc. (1984).
5. Zeev Nehari, Conformal Mappings, Dover Publications Inc. (2003).
6. W. Rudin, Complex Analysis, McGraw Hill, 3rd Edition (2023).