2-Year Master's Program in Mathematics (NEP-2020)

ABSTRACT ALGEBRA

Master's Mathematics (1^{st} Semester) Total Credits: 04

Course Code: MMTHCAA126 (15 hours per credit) Total Marks: 100

Continuous Assessment: Marks 30, Theory: Marks 70 Time Duration: 2½ hrs

Course Learning Outcomes(CLO's): After the completion of this course students shall be able to:

CLO1. Apply the structure theorems for cyclic groups and finite abelian groups.

CLO2. Analyse group homomorphisms, automorphisms, and group actions.

CLO3. Explore group classifications using Sylow theorems and composition series.

CLO4. Gain a working knowledge of polynomial rings and factorization in rings.

UNIT-I

Semigroups and criterion for a semigroup to be a group, Cyclic groups and their generators (finite and infinite), Structure theorem for cyclic groups, Endomorphisms, automorphisms, inner and outer automorphisms, Cauchy's and Sylow's theorems for abelian groups, Groups of symmetries, alternating groups, simple groups, Classification and properties of groups of order six, Simplicity of the alternating group A_n.

UNIT-II

Finite Groups and Group Actions: Conjugacy classes and class equations for groups of orders $p, 2p, p^2, p^3$. Applications of class equations, Sylow theorems for finite groups. Direct product of groups and finite abelian group classification, Normal and subnormal series, Composition series and the Jordan-Hölder theorem, Zassenhaus lemma and Schreier's refinement theorem, Solvable groups: examples and related theorems.

UNIT-III

Rings and Integral Domains: Review of rings and basic definitions, Field of quotients for an integral domain, Embedding of integral domains, Euclidean rings with examples such as $Z[\sqrt{-1}]$, $Z[\sqrt{2}]$, Principal Ideal Rings (PIRs), Unique Factorization Domains (UFDs), Euclidean Domains (EDs), GCD and LCM in rings, Factorization theorem and relationships among EDs, UFDs, and PIRs.

UNIT-IV

Polynomial Rings and Factorization: Polynomial rings and the division algorithm, Irreducible polynomials and primitive polynomials, Rational field and integer monic polynomials, Gauss's Lemma and Eisenstein's Irreducibility Criterion, Cyclotomic polynomials, Contraction of polynomials, Polynomial rings and some basic results of commutative ring theory.

CLO-PLO Mapping Matrix (Strength version)

CLO ⇓ **PLO** ⇒	PLO1	PLO2	PLO3	PLO4	PLO5	PLO6	PLO7	PLO8	Average CLO
MMTHCAA126 .1	3	2	1	3	3	1	1	3	**2.12**
MMTHCAA126 .2	3	3	2	2	2	3	2	2	**2.37**
MMTHCAA126 .3	3	2	2	3	3	2	3	2	**2.5**
MMTHCAA126 .4	3	2	2	2	1	3	1	3	**2.12**
Average PLO	**3**	**2.25**	**1.75**	**2.5**	**2.25**	**2.25**	**1.75**	**2.5**	**2.28**

Recommended Books

1. J. A. Gallian, Contemporary Abstract Algebra, Cengage Learning, USA, 9th Edition, 2015.
2. I.N. Heristein, Topics in Algebra, John Wiley & Sons, 2nd Edition, 1975.
3. P. B. Bhatacharaya and S.K. Jain, Basic Abstract Algebra, Cambridge University Press, 4th Edition, Reprint 2009.
4. J. B. Fragleigh, A First Course in Abstract Algebra, Pearson New International, 2014.
5. K. S. Miller, Elements of Modern Abstract Algebra, Krieger Publishing, 1975.
6. Surjeet Singh and Qazi Zameer-ud-Din, Modern Algebra, Vikas Pub Hou. Pvt Ltd, 8th Edition, 2006.