Bachelors with Mathematics as Major, 8th Semester
MMT822J2: Mathematics/Applied Mathematics:
Metric and Topological Spaces

Credits: 4 THEORY + 2 TUTORIAL **Theory: 60 Hours & Tutorial: 30 Hours**

Course Objectives: To understand basic concepts of Metric and topological spaces, examples, properties and cantor's intersection theorem. Uniform continuity, compact metric spaces and totally bounded sets. Some importannt theorem in topological spaces.

Course Outcome: Students understand basic concepts of metric & topological spaces and their properties, applications and various results to deal with distance functions and other structures.

Theory: 4 Credits

Unit- I

Review of countable and uncountable sets, Schroeder-Bernstein theorem, axiom of choice and its various equivalent forms, definition and examples of metric spaces, open and closed sets, convergence, completeness and Baire's category theorem, and applications to the (1) non-existence of a function which is continuous precisely at irrationals (ii) impossibility of approximating the characteristic of rationals on [0, 1] by a sequence of continuous functions.

Unit – II

Completion of a metric space, Cantor's intersection theorem with examples to demonstrate that each of the conditions in the theorem is essential, Continuous Functions with examples and related results, uniformly continuous mappings with examples and counter examples, extending uniformity continuous maps, Banach's contraction principle with applications to the inverse function theorem in R.

Unit – III

Topological spaces; definition and examples, elementary concepts, open bases and open subbases, Kuratowski's axioms, continuous mappings and their characterizations, pasting lemma, convergence of nets and continuity in terms of nets, lower limit topology, concepts of first countability, second countability, separability and their relationships, counter examples and behavior under subspaces, product topology and weak topology.

Unit – IV

Heine-Borel theorem, Tychnoff's theorem, compactness, sequential compactness and total boundedness in metric spaces, Lebesgue's covering lemma, continuous maps on a compact space, separation axioms T_i $\left(i = 1, 2, 3, 3\frac{1}{2}, 4\right)$ and their permanence properties, connectedness, local connectedness, their relationship and basic properties, connected sets in R, Urysohn's lemma, Urysohn's metrization theorem, Tietize's extension theorem, one point compactification.

Tutorials: 2 Credit

Unit – V

problems related to unit I and unit II.

Unit – VI

problems related to unit III and unit IV.

Recommended Books;

1. G.F.Simmons, Introduction to Topology and Modern Analysis, McGraw Hill Edu. Pvt. Ltd., 2017
2. D Somasundaram & B Choudhary: A first course in Mathematical Analysis, Narosa pub. House, 1996.
3. J. Munkres, Topology-A First Course, Prentice-Hall, New Delhi, 1974.
4. K.D. Joshi, Introduction to General Topology, New Age International Pvt Ltd., 2017.
5. J.L.Kelley, General Topology, Dover Publications, Re-Print, 2017.
6. E T Copson: Metric Spaces, Cambridge University press, 1998.
7. Murdeshwar, General Topology, New Age International Pvt Ltd.,3rd Edition, 2020.
8. S.T. Hu, Introduction to General Topology, Holden Day, 1958.
9. S Shirali and HL Vasudeva: Metric Spaces, Springer, India, 2006th Edition, 2005.
10. S Lipschutz: General Topology, Schaum's Outline, McGraw Hill, 2011.