

Under-Graduate Programme with MATHEMATICS; 7th Semester

MATHEMATICS

MMT726J3: REAL ANALYSIS-II

Theory: 4 Credits (60 Hours)

Credits: 6 (4 (TH) + 2 TUT)

Tutorial: 2 Credits (30 Hours)

Objectives: To make the students understand Inequalities used in real analysis, theorems used in real analysis. To expose students about the basic concepts of Riemann integration, improper integrals and sequence of functions.

Course outcome: The students can handle various problems using such inequalities, Riemann integration, fundamental theorem of integral calculus and other problems of real analysis.

Theory: 4 Credits

Unit- I

Inequalities: Arithmetic-Geometric mean Inequality, Inequalities of Cauchy, Schwarz, Jensen, Holder and Minkowski, Inequalities as the product of Arithmetic means of two sets of positive numbers. Infinite series: Carleman's Theorem, Conditional and Absolute Convergences, Multiplication of Series, Merten's Theorem, Dirichlet's Theorem, Riemann's Rearrangement Theorem.

Unit – II

Riemann Integration: Riemann Integration, Lower and Upper Sums, Refinement of a Partition, Behaviour of Lower and Upper Sums under Refinement, Definition and Existence of the Riemann Integral, Necessary and sufficient condition for R-Integrability of a bounded function, R-Integrability of sum, difference, product and quotient of two functions, R-Integrability of Continuous, Monotone and Discontinuous functions (having finite number of discontinuity) in an Interval, mean value theorem for integrals, the primitive, Fundamental; theorem for Integral Calculus.

Unit – III

Improper Integrals: Integration of unbounded functions with finite limits of integration, Comparison test for convergence, Necessary and Sufficient conditions for Convergence, Cauchy's Test, Infinite range of Integration, Tests for convergence, Absolute convergence, integrand as a product of functions, Abel's and Dirichlet's Test.

Unit – IV

Sequence and Series of Functions: Point-wise and uniform Convergence, Cauchy's criterion for uniform convergence, Mn-Test, Weirstrass M-Test, Abel's and Dirichlet's Test for uniform convergence, Uniform convergence and continuity, R-Integration and differentiation, Weirstrass approximation theorem, Example of Continuous nowhere differentiable function.

Tutorials: 2 Credits

Unit – V

Intermediate value property for the derivative, Young's form of Taylor's Theorem, Generalised second derivative, Abel's limit theorem, Taylor's theorem with integral form of remainder, Bernstein's Theorem for convergence of the Taylor series.

Unit – VI

Riemann – Stieltjes integration, Definition and existence of the integral, condition of integrability, Integral as a Limit of sum, some theorems, Reduction of Riemann–Stieltjes integral to Riemann integral.

References

1. R. Goldberg, Methods of Real Analysis.
2. W. Rudin, Principles of Mathematical Analysis.
3. T. M. Apostol, Mathematical Analysis^{2nd} edition.
4. S. C. Malik and S. Arora, Principles of Mathematical Analysis.
5. L. Royden, Real Analysis.
6. R. G. Bartle and D. R. Sherbert, Introduction to Real Analysis, John Wiley and Sons (Asia) P. Ltd, 2000.
7. S. M. Shah and Saxena, Real Analysis.