

## **Bachelors with Mathematics as Major**

### **7<sup>th</sup> Semester**

#### **MMT722J2: Mathematics/Applied Mathematics: Algebra-II**

**Credits: 4 THEORY + 2 TUTORIAL**

**Theory: 60 Hours & Tutorial: 30 Hours**

**Course Objective:** To instill in the students the fundamental ideas of group theory, preparing them for later topics in more complex abstract algebra and ring theory.

**Course Outcomes:** It enables students to tackle various classical problems in algebra. It also provides a way for the student to take various advanced courses in the future, like ring theory, field theory, commutative algebra, etc.

#### **Theory: 4 Credits**

##### **Unit I**

Automorphisms, Inner and outer automorphisms & related results, Direct sum & Direct product, upto isomorphism, Finite abelian groups, Class Equation. Groups of order  $p$ ,  $2p$ ,  $p^2$ ,  $p^3$ , where  $p$  is a prime number. Cauchy's theorem for finite (finite abelian) groups.

##### **Unit II**

Sylow  $p$ -subgroups, Sylow's theorems & related results, Primary decomposition theorem, Subnormal and normal series, Composition series, Jordan Holder theorem, Zassenhaus lemma, Schreier's refinement theorem, Solvable groups

##### **Unit III**

Prime & Maximal ideals in a ring & related results, Integral domain, Euclidean rings with examples such as  $Z[\sqrt{-1}]$ ,  $Z[\sqrt{2}]$ , Principal ideal rings(PIR), Unique factorization domains(UFD) and Euclidean domain, Greatest common divisor (GCD) & Least common multiple (LCM) in rings, Relationship between P.I.R.'s & U.F.D.'s

##### **Unit IV**

Polynomial rings, Division algorithm in  $F[X]$  (where  $F$  is a field), Fundamental theorem of algebra, Reducible and irreducible polynomials, Content of a polynomial and primitive polynomial, Gauss's lemma, Eisenstein's irreducibility criteria.

#### **Tutorial: 2 Credits**

##### **Unit V**

Number of abelian groups (upto isomorphism) of a finite order, Number of non-isomorphic abelian groups of order  $n$ , Problems related to counting of automorphisms & inner automorphisms in a group, Simple groups, Number of  $p$ -sylow subgroups, problems on subnormal, composition series and solvable groups.

##### **Unit VI**

Prime, reducible & irreducible elements in a ring, Units & non-units in a ring, Reducibility test of polynomials for degrees 2 and 3, Reducibility and irreducibility of polynomials over different fields.

#### **Recommended Books:**

1. I. N. Herstein, Topics in Algebra, Wiley, 2016
2. P.B. Bhattachariya, S.K. Jain, S. R. Nagpaul, Basic Abstract Algebra, Cambridge, 1994
3. J. B. Fraleigh, A First Course in Abstract Algebra, Pearson Education India.
4. Joseph Gallian, Abstract Algebra, Narosa Publishers, New Delhi, 1999.
5. K. S. Miller, Elements of Modern Abstract Algebra, Cambridge University Press.
6. Surjeet Singh and Qazi Zameer-ud-Din, Modern Algebra, Vikas Publishing House Private Limited, 2021
2. V.K. Khanna & S.K. Bhambri, A Course in Abstract Algebra, Vikas Publishing.