

Bachelors with Mathematics as Major
6th Semester

MMT622J3: Mathematics/Applied Mathematics: ADVANCED CALCULUS

Credits: 4 THEORY + 2 TUTORIAL

Theory: 60 Hours & Tutorial: 30 Hours

Course Objectives: To enable students to understand functions of several variables. To make students able to understand the techniques for solving double and triple integrals.

Course Outcome: After the completion of this course, students shall be able to solve double and triple integrals. To evaluate problems using Green's theorem, surface integrals for solution of integrals.

Theory: 4 Credits

Unit- I

Functions of several variables. Limit and continuity of a function of two variables. Algebra of limits. Partial derivative of function of two variables. Mean value theorem. Sufficient condition for continuity. Differentiability of functions of two variables, Sufficient condition for differentiability for functions of two variables. Explicit and implicit functions.

Unit – II

Algebra of differentiable functions. Concept of partial derivatives of higher order. Change in the order of partial derivatives. Sufficient condition for equality of f_{xy} and f_{yx} . Young's theorem and Schwarz's theorem. Functions of functions. Derivative of composite functions (the chain rule), total differentiability.

Unit – III

Jacobians: Some properties and applications in two and three variables. Change of variables, Taylor's theorem. Extreme values: Maxima and minima of functions of two variables, sufficient conditions for $f(x, y)$ to have an extreme value. Lagrange's method of undetermined multipliers. Volume of the greatest rectangular parallelepiped that can be inscribed in the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ is $\frac{8ab}{3\sqrt{3}}$. The length of the axis of the section of the ellipsoid. $\sum \frac{x^2}{a^2} = 1$ by the plane $lx + my + nz = 0$ are the roots of the quadratic equation $\frac{l^2 a^2}{r^2 - a^2} + \frac{m^2 b^2}{r^2 - b^2} + \frac{n^2 c^2}{r^2 - c^2} = 0$ in r^2 . If $xyz = abc$, the minimum value of $bcx + cay + abz$ is $3abc$.

Unit – IV

Double integrals: Integration over a rectangle. Condition of integrability. Calculation of double integral over a rectangle. Integrability over a bounded domain. Calculation of double integral over a closed domain. Green's theorem in R^2 . Integration in R^3 . Line, surface and volume integrals. Calculation of triple integrals (simple cases).

Tutorials: 2 Credits

Unit – V

Applications of Mean value theorem, Young's theorem, Schwarz's theorem for functions of two variables.

Unit – VI

Applications of change of variables, Green's theorem, Surface integrals.

Recommended Books;

1. Mathematical Analysis, S. C. Malik and Savita Arora, 6th Edition, New Age, 2021.
2. Advanced Calculus, David V. Widder, 2nd Edition, PHI, New Delhi, Pvt Ltd.
3. Sudhir R. Ghorpade, B.V. Limaye: A Course in Multivariable Calculus and Analysis, Springer International Edition, 2010.
4. Advanced Calculus, Kochar Chopra, 2020
5. S. Dineen, Functions of two variable, Chapman and Hall, 1995.
6. David Widder, Advanced Calculus, Prentice Hall of India, 2017.
7. T.M Apostol, Mathematical Analysis, Narosa Publication house, 2002.