Bachelors with Mathematics as Major

5th Semester

MMT522J1: Mathematics/Applied Mathematics: Algebra-I

Credits: 3 THEORY + 1 TUTORIAL **Theory: 45 Hours & Tutorial: 15 Hours**

Course Objective: To introduce students towards basic concepts of algebraic structures, viz groups and rings. (ii) to identify various properties associated with the groups and rings. (iii) to expose students towards advanced mathematics, such as advanced abstract algebra and communtative ring theory.

Course Outcomes: After the completion of this course, students shall be able to (i) understand group through symmetries in nature and can identify patterns. (ii) shall be able to apply these concepts in linear classical groups to solve problems arising in physics, computer science, economics and engineering etc.

Theory: 3 Credits

Unit I

Binary composition, equivalence relation and equivalence class. Groups: Definition and examples, Finite & Infinite groups, abelian & non-abelian groups. Properties of groups: uniqueness of identity/ inverse and cancellation laws. Subgroups and Cosets. Criterion of subgroups, order of an element, Lagrange's theorem. Cyclic groups: Definition & examples, cyclic groups are abelian, order of a cyclic group is equal to the order of its generator.

Unit II

Normal subgroups: Definition & examples, criterion of normal subgroups, product of subgroups, Quotient groups. Homomorphism, kernel of a homomorphism, Fundamental theorem of homomorphism, Isomorphism theorems (statements only), Normalizer of an element, Centre of a group. Permutation groups and Cayley's theorem. Definitions and examples of symmetric groups and alternating groups.

Unit III

Rings: Definition & Examples, elementary properties of rings, rings with & without zero divisors, Integral domains & skew fields. Definitions & examples of Fields, Subrings, Ideals, Quotient rings and Boolean rings. A finite integral domain is a field. Ring homomorphism, fundamental theorem of homomorphism & ring isomorphism. Prime, Maximal and Principal ideals (Definitions & examples only).

Tutorial: 1 Credit

Unit IV

Examples of monoids, semigroups, abelian/non-abelian, cyclic/non-cyclic groups, computing generators in a cyclic group, Structure theorem for cyclic groups (statement only), concept of even and odd permutation. Examples of commutative & non-commutative rings, Left & right ideal of a ring, relation between ideals & subrings, sum & product of ideals.

Recommended Books:

1. I. N. Herstein, Topics in Algebra, John Wiley, 1975.
2. Joseph Gallian , Abstract Algebra, Narosa Publishers, New Delhi, 1999.
3. M. Artin, Algebra, Pearson Education India, 2011.
4. D. S. Dumit and R. M. Foote, Abstract Algebra, John Wiley, 2003
5. P.B. Bhattachariya, S.K. Jain, S. R. Nagpaul, Basic Abstract Algebra, Cambridge University Press, 1994
6. Surjeet singh and Qazi Zameeruddin, Modern Algebra, S Chand And Company Ltd, 2021