

Bachelors with Mathematics as Major

3rd Semester

MMT322J: Mathematics/Applied Mathematics: Theory of Matrices

Credits: 4 THEORY + 2 TUTORIAL

Theory: 60 Hours & Tutorial: 30 Hours

Course Objectives: (i) To understand matrix theory as a tool to solve various real life problems.

(ii) To make students aware about the properties and applications of matrices.

Course Outcome: After the completion of this course, students shall be able to (i) apply techniques of matrix theory to solve real life problems (ii) use matrix techniques in coding theory and cryptography (iii) use eigenvalues to find the stability of various systems.

Theory: 4 Credits

Unit –I

Generalization of reversal law of transpose, Hermitian and skew- Hermitian matrices, Representation of a square matrix as $P + iQ$, where P and Q are both Hermitian. Adjoint of a matrix, For a square matrix A, $A(\text{adj}A) = (\text{adj}A)A = |A|I$. Commutative and associative laws in matrix operations. Necessary and sufficient condition for a square matrix to be invertible. Generalization of reversal law for the inverse of matrices under multiplication.

Unit –II

The operation of transposing and inverting are commutative, trace of a matrix, trace of $AB =$ trace of BA and its generalization. Partitioning of matrices, Matrix polynomials and Characteristic equation of a square matrix. Cayley-Hamilton theorem, Eigen values and Eigen vector, minimal equation of a matrix.

Unit –III

Rank of a matrix. Elementary row (Column) transformations of a matrix do not alter its rank, rank of a matrix by elementary transformations, reduction of a matrix to the normal form, Elementary matrices. Every non- singular matrix is a product of elementary matrices, employment of only row (column) transformations. Rank of product of two matrices. Linear combination, linear dependence and linear independence of Row (Column) vectors, the columns of a matrix A are linearly dependent iff there exists a vector $X \neq 0$ such that $AX=0$. The columns of a matrix A of order $m \times n$ are linearly dependent iff rank of $A < n$. The matrix A has rank r if and only if it has r linearly independent columns and any s-columns ($s > r$) are linearly dependent (analogous results for rows).

Unit –IV

Linear homogeneous and non- homogeneous equations, the equation $AX=0$ has a non-zero solution if and only if rank of $A < n$, the number of its columns, the number of linearly independent solutions of the equation $AX=0$ is $n-r$, where r is the rank of matrix A of order $m \times n$, the equation $AX=B$ is consistent if and only if two matrices A and $[A: B]$ are of the same rank. Inner product of two vectors, length of a vector, normal vectors, Orthogonal and Unitary matrices, A matrix P is orthogonal (Unitary) if and only if its column vectors are normal and orthogonal in pairs.

Tutorial: 2 Credits

Unit –V

Problems based on Hermitian and skew-Hermitian and inverse of matrices. Problems on characteristic roots and characteristic polynomials. Applications of Cayley Hamilton theorem for the inverse of a matrix.

Unit –VI

System of equations and their solutions, examples of determination of orthogonal matrices and examples of system of homogenous and non-homogenous equation having unique, infinite and no solution.

Books Recommended:

1. Shanti Narayan, A textbook of Matrices, Schaum S. Chand and Company, 1957.
2. A. Aziz and NA Rather and BA Zargar, Elementary Matrix theory, Kapoor Book Depot, Srinagar, 2007.
3. K. Hoffman and R. Kunze, Linear Algebra, Pearson Education, 2018.
4. S. Lipschutz & M. Lipson, Linear Algebra, Schaum's outline series, Tata McGraw-Hill, 4th Edition 2009.