NON-LINEAR ANALYSIS

Course Code: **MM24408DCE** Total Credits: **04**
Semester: **MA/M.Sc. 4th Semester** Total Marks: **100**
Continuous Assessment: **Marks 20,** Theory: **Marks 80** Time Duration: **2½ hrs**

Course Objectives: To inculcate the students about various methods to solve problems involving the homogeneous and non-homogeneous operators. Projection mappings, bilinear forms and variational inequalities shall be taught in the course.
Course Outcome: After the completion of this course, students shall be able to apply operators arising in various real life situations.

……

Unit - I
Convex Sets, best approximation properties, topological properties, separation, non-expansive operators, projectors onto convex sets, fixed points of non-expansive operators, averaged non-expansive operators, Fejer monotone sequences, convex cones, generalized interiors, polar and dual cones, tangent and normal cones, convex functions, variants, between linearity and convexity, uniform and strong convexity, quasiconvexity.

Unit - II
Gateaux Derivative, Frechet Derivative, lower semicontinuous convex functions, subdifferential of convex functions, directional derivatives, characterization of convexity and strict convexity, directional derivatives and subgradients, Gateaux and Frechet differentiability, differentiability and continuity

Unit - III
Monotone operators, strong notions of monotonicity such as para, cyclic, strict, uniform and strong monotonicity, maximal monotone operator and their properties, bivariate functions and maximal monotonicity, Debrunner-Flor theorem, Minty theorem, Rockfeller's cyclic monotonicity theorem, monotone operators on R.

Unit - IV
Reisz-Representation theorem, projection mappings and their properties, characterization of projection onto convex sets and their geometrical interpretation, Billinear forms and its applications, Lax-Milgram lemma, minimization of functionals, variational inequalities, relationship between abstract minimization problems and variational inequalities, Lions Stampacchia theorem for existence of solution of variational inequality.

Recommended Books:

1. H. H. Bauschke and P. L. Combettes, Convex Analysis and Monotone Operator Theory in Hilbert Spaces, Springer New York, 2011.
2. D. Kinderlehrer and G. Stampacchia, An Introduction to Variational Inequalities and Their Applications, Academic Press, New York, 1980.
3. A. H. Siddiqi, K. Ahmed and Manchanda, P. Introduction to Functional Analysis with Applications, Anamaya Publishers, New Delhi-2006.
4. I. Ekland and R. Temam, Convex Analysis and Variational Problems, W.Takahashi, Nonlinear Functional Analysis, North-Holland Publishing Company-Ammsterdam, 1976.
5. M. C. Joshi and R. K. Bose, Nonlinear Functional Analysis and its Applications, Willey Eastern Limited, 1985.