DIFFERENTIAL GEOMETRY

Course No: **MM24402CR** Total Credits: **04**
Semester: **MA/M.Sc. 4th Semester** Total Marks: **100**
Continuous Assessment Marks: **20**, Theory Marks: **80** Time Duration: **2½ hrs**

Course Objectives: To inculcate the students about the geometric properties of curves, surfaces and their higher-dimensional analogs using the methods of calculus. Christoffel symbols, regular surfaces, geodesics and isometry between surfaces.

Course Outcome: After the completion of this course, the students shall be able to understand basic properties of curves and surfaces. They shall be able to use various properties of curves and surfaces in the geometry of the nature.

UNIT -I

Curves: differentiable curves, regular points, parameterization and reparameterization of curves, unit speed curves, arc-length is independent of parameterization, plane curves, curvature of plane curves, osculating circle. Space curves, Serret-Frenet frame, fundamental theorem of space curves. Characterization of helices and curves on sphere in terms of their curvature and torsion, evolutes and involutes of space curves, Isoperimetric inequality, Four vertex theorem.

UNIT -II

Surfaces: regular surfaces with examples, coordinate charts or curvilinear coordinates, change of coordinates, tangent plane at a regular point, normal to the surface, orientable surface, differentiable mapping between regular surfaces and their differential, first fundamental form, line element, invariance of a line element under change of coordinates, angle between two curves, condition of orthogonality of coordinate curves, area of bounded region, invariance of area under change of coordinates.

UNIT -III

Curvature of a Surface: normal curvature, principal curvatures and principle directions, classification of points with prescribed principal curvatures, Euler's formula, Gauss map and its differential, second fundamental form, normal curvature in terms of second fundamental form. Meunier theorem, Gaussian and mean curvature, Hilbert's theorem and its applications. Weingarten equation, surface of revolution, surfaces with constant positive or negative Gaussian curvature, Gaussian curvature in terms of area, line of curvature, Rodrigue's formula for line of curvature, equivalence of surfaces, isometry between surfaces, local isometry, and characterization of local isometry.

UNIT -IV

Christoffel symbols, expressing Christoffel symbols in terms of metric coefficients and their derivative, Theorema egrerium. Gauss equations and Manardi Codazzi equations for surfaces, fundamental theorem for regular surface. (Statement only). Geodesics: geodesic curvature, geodesic curvature is intrinsic, equations of geodesic, geodesic on sphere and pseudo sphere, geodesic as distance minimizing curves. Gauss-Bonnet theorem (statement only), geodesic triangle on sphere, implication of Gauss-Bonnet theorem for geodesic triangle.

Recommended Books:

1. M.P. Do Carmo, Differential geometry of curves and surfaces, Dover Publications, 2nd Ed. 2016.
2. W. Klingenberg, A course in Differential Geometry, Spring Verlag, Edition, 2013.
3. C. E. Weatherburn, Differential Geometry of Three dimensions, Cambridge Univ. Press, 2016.
4. T. Willmore, An Introduction to Differential Geometry, Oxford University Press, 1997.
5. Sebastian Montiel, Antonio Ros, Curves and surfaces, AMS, 2nd Edition, 2009.