ABSTRACT MEASURE THEORY

Course No: **MM24306DCE** — Total Credits: **04**
Semester: **M.A/M.Sc 3rd Semester** — Total Marks: **100**
Continuous Assessment Marks **20**, Theory Marks: **80** — Time Duration: **2½ Hrs**

Course objectives: To extend the concept of measure to abstract spaces for various measures in order to obtain corresponding analogs of various results of Lebesgue measure which involves general theory of measure spaces and measurable functions, Lebesgue measure on the real line, the sigma algebras, a brief introduction to L_p −spaces and related topics.

Course Outcomes: The course shall help the students in developing understanding of measure theory concepts, abstract integration, and their applications in various mathematical contexts.

UNIT-I

Semi-ring, ring, algebra and σ −algebra of sets, measures on semi-rings, outer measure associated with a set function and basic properties, measurable sets associated with an outer measure as a σ − algebra , outer measure induced by a measure, non measurable sets.

UNIT-II

Finite and σ- Finite measure spaces, Measurable sets of finite measure space, Improper Riemann integral as a Lebesgue integral, calculation of some improper Riemann integrable functions, approximation of integrable functions, Riemann Lebesgue lemma.

UNIT-III

Product measures and product σ −algebra, measurable rectangles, monotone class and elementary sets, expressing a double integral as an iterated integral, examples of non-integrable functions whose iterated integrals exist (and are equal), Integration on product spaces, Fubini theorem.

UNIT-IV

For $f \in L_1[a,b], F' = f a.e.$ on $[a,b]$, if f is absolutely continuous on (a,b) with $f(x) = 0 a.e.$, then f is constant. Characterization of an absolutely continuous function as an indefinite Lebesgue integral. Non-Lebesgue integrability of $x^2 sin(1/x^2)$ on $[0,1]$. Fundamental theorem of calculus for the Lebesgue integral. A brief introduction to L_p spaces. Holder's and Minkowski's inequalities.

Recommended Books:

1. C. D. Aliprantis and O. Burkinshaw, Principles of Real Analysis, Academic Press Inc. 2nd Edition (1990).
2. Goldberg , R, Methods of Real Analysis, Oxford and IBH Publishing Company (2020).
3. T. M. Apostol, Mathematical Analysis, Narosa (2002).
4. Royden, L, Real Analysis, Pearson Education India, 4th Edition (2015).
5. Chae, S.B., Lebesgue Integration, Springer Verlag, 2nd Edition (1995).
6. Rudin, W., Principles of Mathematical Analysis, McGraw Hill (2023)
7. Barra, De. G., Measure theory and Integration, New Age International Publishers, 3rd Edition (2022).
8. Rana , I. K., An Introduction to Measure and Integration, Narosa Publications.