FUNCITONAL ANALYSIS-I

Course No: **MM24303CR** Total Credits: **04**
Semester: **MA/M.Sc. 3rd Semester** Total marks: **100**
Continuous Assessment Marks: **20**, Theory Marks: **80** Time Duration: **2½ hrs**

Course Objectives: To familiarize with the basic tools of Functional Analysis involving normed spaces, Banach spaces, inner product spaces, Hilbert spaces and their geometric properties, transformation of bounded linear operators from one space to another.

Course Outcomes: After studying this course the student will be able to distinguish between Banach spaces and Hilbert spaces, decompose a Hilbert space in terms of orthogonal complements, check totality of orthonormal sets and sequences, represent a bounded linear functional in terms of inner product and develop understanding to the theory of functional spaces, linear operators and their application in various Mathematical and scientific contexts.

UNIT-I

Banach Spaces: definition and examples, subspaces, quotient spaces, continuous linear operators and their characterization, completeness of the space L(X,Y) of bounded linear operators (and its converse), incompleteness of C[a, b] under the integral norm, finite dimensional Banach spaces, equivalence of norms on finite dimensional space and its consequences, dual of a normed linear space, , duals of l_p^n, C_o , l_p ($p\geq1$), C[a, b].

UNIT-II

Hahn Banach theorem (extension form) and its applications, Uniform boundedness, principle and weak boundedness, dimension of an ∞-dimensional Banach space, conjugate of a continuous linear operator and its properties, Banach-Steinhauss theorem, open mapping and closed graph theorems, counterexamples to Banach-Steinhauss, open mapping theorem and closed graph theorems for incomplete domain and range spaces, separable Banach spaces and the separability of some concrete Banach spaces (C_o, C[0,1], l_p, $p\geq1$), reflexive Banach Spaces, closed subspace and the dual of a reflexive Banach space, examples of reflexive and non-reflexive Banach spaces.

UNIT-III

Hilbert spaces: definition and examples, Cauchy's Schwartz inequality, parallogram law, orthonormal (o.n) systems, Bessel's inequality and Parseval's Identity for complete orthonormal systems, Riesz-Fischer theorem, Gram Schmidt process, o.n basis in separable Hilbert spaces.

UNIT-IV

Projection theorem, Riesz Representation theorem, counterexample to the projection theorem and Rieszrepresentation theorem for incomplete spaces, Hilbert property of the dual of a Hilbert space and counter examples of incomplete inner product spaces, reflexivity of Hilbert space, adjoint of a Hilbert space operator, weak convergence and Bolzano-Weirstrass property in Hilbert Spaces, normal and unitary operators, finite dimensional spectral theorem for normal operators**.**

Recommended Books:

1. B. V. Limaya, Functional Analysis, New Age International Pvt. Ltd; 3rd edition, 2014.
2. C. Goffman & G. Pedrick, A First Course in Functional Analysis, American Mathematical Society; 2nd edition, 2017.
3. L.A. Lusternick & V.J. Sobolov, Elements of Functional Analysis, Hindustan Publishing Corporation (India); 3rd edition, 2020.
4. J.B. Conway, A Course in Functional Analysis, Springer, 4th Edition, 1994.