COMPLEX ANALYSIS-II

Course No: **MM24302CR** Total Credits: **04**
Semester: **M.A/M.Sc 3rd Semester** Total Marks: **100**
Continuous Assessment: **Marks 20, Theory Marks: 80** Time Duration: **2½ Hrs**

Objectives: The main aim of this course is to solve definite integrals by the method of Contour integration, bounds for the range of the analytic functions and concept of analytic continuation of a power series in order to have an understanding about the behavior of functions in complex domain.

Course Outcomes: The students shall be able to explore advanced integration techniques in the complex plane, including contour integrals, residue theorem applications, and evaluation of real integrals. This will help in understand the concept of analytic continuation and its applications in extending the domain of validity of complex functions.

UNIT -I

Calculus of Residues, Cauchy's residue theorem, evaluation of integrals by the method of residues, Parseval's Identity, branches of many-valued functions with special reference to $arg(z)$, $log\ z$ and z^n, Blashke's theorem.

UNIT -II

Poisson integral formula for circle and half plane, Poisson-Jensen formula, Estermann's uniqueness theorem, Carlemann's theorem and the uniqueness theorem associated with it, Hadamard's three circle theorem, $logM(r)$ and $logI_2(r)$ as convex function of $logr$, theorem of Borel and Carateodory.

UNIT -III

Power series: Cauchy-Hadamard formula for the radius of convergence, Picard's theorem on power series: If $a_n > a_n + 1$ and $liman = 0$, then the series $\sum a_n z^n$ has radius of convergence equal to 1 and the series converges for $|z| = 1$ except possibly at z =1, a power series represents an analytic function within the circle of convergence, Hadamard - Pringsheim theorem, the principle of analytic continuation, uniqueness of analytic continuation, power series method of analytic continuation, functions with natural boundaries e.g., $\sum z^{n!}, \sum z^{2^n}$. Schwarz reflection principle.

UNIT -IV

Functions with positive real part, Borel's theorem, univalent functions, area theorem, Bieberbach's conjecture (statement only) and Koebe's ¼ theorem. Space of analytic functions, Bloch's theorem, Schottky's theorem, a - points of an analytic function, Picard's theorem for integral functions, Landau's theorem.

Recommended Books:

1. Alfhors, Complex Analysis, McGraw Hill (2000).
2. E. C. Tichtmarsh, Theory of Functions, Oxford University Press, 2nd Edition (1976).
3. J. B. Conway, Functions of a Complex Variable-I, Springer, 2nd Edition (1995).
4. Richard Silverman, Complex Analysis, Dover Publications Inc. (1984).
5. Zeev Nehari, Conformal Mappings, Dover Publications Inc. (2003).
6. W. Rudin, Complex Analysis, McGraw Hill, 3rd Edition (2023).