THEORY OF NUMBERS -II

Course No: **MM24205DCE** Total Credits: **04**
Semester: **M.A/M.Sc 2nd Semester** Total Marks: **100**
Continuous Assessment: **Marks 20, Theory Marks: 80** Time Duration: **2½ Hrs Course**

Course objectives: To identify certain number theoretic functions and their properties in order to study real number system in depth for their applications.

Course Outcomes: It typically focus on more advanced topics in number theory, including deeper exploration of properties of integers, prime numbers, and applications.

UNIT -I
Integers belonging to a given exponent (mod p) and related results, converse of Fermat's theorem; If $d|p-1$, the congruence $x^d \equiv 1(modp)$ has exactly d-solutions; If any integer belongs to $t(modp)$, then exactly $\phi(t)$ incongruent numbers belong to $t(modp)$, primitive roots, there are $\phi(p-1)$ primitive roots of an odd prime p, any power of an odd prime has a primitive root, the function λ (m) and its properties, $a^{\lambda(m)} \equiv 1(modm)$, where $(a,m) = 1$. There is always an integer which belongs to λ(m) (mod m), primitive λ -roots of m, the numbers having primitive roots are $1,2,4, p^\alpha, 2p^\alpha$ where p is an odd prime.

UNIT -II
Quadratic residues, Euler criterion, the Legendre symbol and its properties, Lemma of Gauss, the law of a quadratic reciprocity, characterization of primes of which 2,-2, 3,-3, 5, 6 and 10 are quadratic residues or non residues, Jacobi symbol and its properties, the reciprocity law for Jacobi symbol.

UNIT -III
Number theoretic functions, some simple properties of $\tau(n), \sigma(n), \phi(n) and \mu(n)$. Mobius inversion formula. Perfect numbers. Necessary and sufficient condition for an even number to be perfect, the function $[x]$ and its properties, average order of magnitudes of $\tau(n), \sigma(n), \phi(n)$, Farey fractions, rational approximation.

UNIT -IV
Simple continued fractions, application of the theory of infinite continued fractions to the approximation of irrationals by rationals, Hurwitz theorem, Relation between Riemann Zeta function and the set of primes, characters, the L-Function $L(S, \chi)$ and its properties, Dirichlet's theorem on infinity of primes in an arithmetic progression.

Recommended Books

1. W. J. Leveque Topics in Number Theory, Vol. I-II Addition WPC, INC.
2. I. Niven and H.S. Zuckerman, An introduction of the Theory of Numbers, Wiley 5th Edition (2008)
3. T.M Apostal, Analytic Number Theory, Springer International, Narosa (1998).
4. G.H Hardy and Wright, An introduction to the theory of Numbers, Oxford University Press, 6th Edition (2008).
5. E. Landau, An Elementary Number Theory, American Mathematical Society, 1958.