REAL ANALYSIS - II

Course No: **MM24202CR** Total Credits: **04**
Semester: **M.A/M.Sc 2nd Semester** Total Marks: **100**
Continuous Assessment: **Marks 20, Theory Marks: 80** Time Duration: **2½ Hrs Course**

Course objectives: To provide the students the notions of length, area and volume with respect to different measures viz., Lebesgue and Borel measure in order to overcome problems arising from Riemann Integration.

Course Outcomes: This course will develop a thorough understanding of real numbers, their properties, and the axiomatic structure of the real number system. This will help the students to recognize the foundational role of real analysis in advanced mathematical areas such as functional analysis, complex analysis, and differential equations.

UNIT -I

Measure theory: definition of outer measure and its basic properties, outer measure of an interval as its length, countable additivity of the outer measure, Borel measurable sets and Lebesgue measurability of Borel sets, Cantor set, existence of non- measurable sets and of measurable sets which are not Borel, outer measure of monotonic sequences of sets.

UNIT -II

Measurable functions and their characterization, algebra of measurable functions, Stienhauss theorem on sets of positive measure, Ostroviski's theorem on measurable solution of $f(x+y) = f(x) + f(y), x, y \in R$, convergence a.e., convergence in measure and almost uniform convergence, their relationship on sets of finite measure, Egoroff's theorem.

UNIT -III

Lebesgue integral of a bounded function, equivalence of L −integrability and measurability for bounded functions, Riemann integral as a Lebesgue integral, basic properties of Lebesgue – integral of a bounded function, fundamental theorem of calculus for bounded derivatives, necessary and sufficient condition for Riemann integrability on $[a, b]$, L −integral of non- negative measurable functions and their basic properties, Fatou's lemma and monotone convergence theorem, L −integral of an arbitrary measurable function and basic properties, dominated convergence theorem and its applications.

UNIT -IV

Absolute continuity and bounded variation, their relationships and counter examples, indefinite integral of an L-integrable function and its absolute continuity, necessary and sufficient condition for bounded variation, Vitali's covering lemma and a. e., differentiability of a monotone function f and $\int f' \leq f(b) - f(a)$.

Recommended Books:

1. R. Goldberg, Methods of Real Analysis, Oxford and IBH Publishing (2020).
2. G. De. Barra, Measure theory and Integration, New Age International Private Limited, 3rd Edition, 2022.
3. I. K. Rana , An Introduction to Measure and Integration, Narosa (2007).
4. W. Rudin, Principles of Mathematical Analysis, McGraw Hill, Standard Edition (2023).
5. Chae, Lebesgue Integration, Springer, 2nd Edition (1995).
6. T. M. Apostol, Mathematical Analysis, Narosa 2nd Edition (2002).