THEORY OF MATRICES

Course No: **MM24105DCE** Total Credits: **04**
Semester: **M.A/M.Sc 1st Semester** Total Marks: **100**
Continuous Assessment: **Marks 20**, **Theory Marks: 80** Time Duration: **2½ Hrs**

Course objectives: To inculcate the students to understand and apply the techniques of matrices like linear transformations from a vector space to itself such as reflection, rotation and sharing to solve multivariate problems arising in different disciplines of science and technology.

Course Outcomes: The Theory of Matrices course provides students with a comprehensive understanding of the fundamental principles and applications of matrices. Upon successful completion of this courses students will be able to classify matrices based on their dimensions (e.g., square, rectangular) and special properties (e.g., symmetric, diagonal, orthogonal). Students will understand the significance of eigenvalues, eigenvectors, and determinants in characterizing matrices.

UNIT-I

Eigenvalues and eigenvectors of a matrix and their determination, similarity of matrices, two similar matrices have the same eigenvalues, algebraic and geometric multiplicity of eigenvalues, Mutual relationship between eigenvalues and the corresponding eigenvectors, any system of eigenvectors corresponding to distinct eigenvalues of a matrix are linearly independent, orthogonal reduction of real symmetric matrices, unitary reduction of a Hermitian matrix.

UNIT-II

Determination of diagonal matrices, the necessary and sufficient conditions for a square matrix of order n to be similar to a diagonal matrix. Orthogonal diagonalization of symmetric matrices, triangular form over $\mathbb{C}$ and $\mathbb{R}$, Schur's theorem, normal matrices. Norms on spaces of matrices: Euclidean, Cartesian and taxicab norms, Schur's inequality: If A is a square matrix of order n having eigenvalues $\lambda_k, 1 \leq k \leq n$, then $\sum|\lambda_k|^2 \leq \sum|a_{ij}|^2$. If A is a square matrix of order n having singular values $\sigma_k, 1 \leq k \leq n$, then $|tr(A)| \leq \sum\sigma_k$. If A is a normal matrix and $\lambda_k, 1 \leq k \leq n$ are its eigenvalues and μ is the eigenvalue of the perturbed matrix $A + \delta A$, then for some $i, \| \mu - \lambda_i \| \leq \| \delta A \|$.

UNIT-III

Quadratic forms: Kroneckers and Lagranges reduction, reduction by orthogonal transformation of real quadratic forms, geometrical applications, necessary and sufficient condition for a quadratic form to be positive definite, If the quadratic form $X^{'}AX$ is positive semidefinite and if x_1 actually appears in the form , then $a_{ii} > 0$ rank, A real symmetric matrix A is positive definite if and only if A^{-1} exists and is positive definite and symmetric, Euler's theorem, Hessian matrix, rank, index and signature of a quadratic form. If $A = [a_{ij}]$ is a positive definite matrix of order n, then $|A| \leq \prod a_{ii}$

UNIT IV

Gram matrices: the Gram matrix BB^t is always positive definite or positive semi-definite, Hadamard's inequality, If $B = [b_{ij}]$ is an arbitrary non- singular real square matrix of order n, then $|B| \leq \prod[\sum_{k=1}^{n} b_{ik}]$. Fundamental scalar function ϕ_k, For any two matrices A and B, $\phi_k(AB) = \phi_k(BA)$, the infinite n −fold integral $I_n = \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} e^{X\,AX} dX$, where $dX = dx_1 \cdots dx_n$. If A is a positive definite matrix, then $I_n = \pi^{n/2}/\sqrt{|A|}$. If A and B are positive definite matrices, then $|\lambda A + (1 - \lambda B)| \leq |A|^{\lambda}|B|^{1-\lambda}$ for $0 \leq \lambda \leq 1$, perturbation of roots of polynomials, companion matrix, Hadamard's theorem, Gerisgorian Disk theorem, Taussky's theorem.

Recommended Books:

1. Richard Bellman, Introduction to Matrix Analysis, McGraw Hill Inc. USA, 2nd Edition 1970.
2. Franz E. Hohn, Elementary Matrix Algebra, Dover Publications, 3rd Edition, 2103.
3. Rajendra Bhatia, Matrix Analysis, Springer.
4. Fuzhen Zhang, Matrix Theory: Basic Results and Techniques (Universitext), Springer, 2nd Editon, 2011.
5. H. Fumio, Petz Dene, Introduction to matrix Analysis and Applications, TRIM (2014).