

## REAL ANALYSIS - I

Course No: **MM24102CR**  
Semester: **MA/M.Sc. 1<sup>st</sup> Semester**  
Continuous Assessment: **Marks 20, Theory: Marks 80**

Total Credits: **04**  
Total Marks: **100**  
Time Duration: **2½ hrs**

**Course objectives:** To study the behavior and properties of real numbers, sequences and series of real numbers and real valued functions and generalized integration in order to tackle day today life problems arising from physical phenomenon.

**Course Outcomes:** The outcome of this course typically includes a solid understanding of fundamental concepts in real analysis, properties and applications of real analysis in convergence of series and sequences.

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### **UNIT-I**

Integration: Definition and existence of Riemann–Stieltje’s integral, behavior of upper and lower sums under refinement, necessary and sufficient conditions for RS-integrability of continuous and monotonic functions, reduction of an RS-integral to a Riemann integral, basic properties of RS-integrals, differentiability of an indefinite integral of continuous functions, the fundamental theorem of calculus for Riemann integrals.

### **UNIT-II**

Improper integrals: integration of unbounded functions with finite limit of integration, comparison tests for convergence, Cauchy’s test, infinite range of integration, absolute convergence, integrand as a product of functions, Abel’s and Dirichlet’s test.

Inequalities: arithmetic-geometric means equality, inequalities of; Cauchy Schwartz, Jensen, Holder & Minkowski, inequality on the product of arithmetic means of two sets of positive numbers.

### **UNIT-III**

Infinite series: Carleman’s theorem, conditional and absolute convergence, multiplication of series, Merten’s theorem, Dirichlet’s theorem, Riemann’s rearrangement theorem. Young’s form of Taylor’s theorem, generalized second derivative, Bernstein’s theorem and Abel’s limit theorem.

### **UNIT-IV**

Sequences and series of functions: point wise and uniform convergence, Cauchy criterion for uniform convergence,  $M_n$  –test, Weierstrass M-test, Abel’s and Dirichlet’s test of uniform convergence, uniform convergence and continuity, R-integration and differentiation, Weierstraa approximation theorem, examples of continuous nowhere differentiable functions.

### **Recommended Books:**

1. R. Goldberg, Methods of Real Analysis, Oxyford and IBH Publishing, 2020.
2. W. Rudin, Principles of Mathematical Analysis, McGraw Hill Edu, 3<sup>rd</sup> Edition, 2017.
3. J. M. Apostol, Mathematical Analysis, Narosa Publishers, 2002.
4. S.C. Saxena and SM Shah : Introduction to Real Variable Theory, Intext Educational Publishers San Francisco, 1972.
- A. J. White, Real Analysis: An Introduction, Addison-Wesley; First Edition, 1968.
5. S.C.Malik and Gupta, Real Analysis, New Age International, 2021.