Post Graduate Department of Mathematics
University of Kashmir, Srinagar – 190006

Pre-Ph.D Course work syllabi-2017

Scheme for the course work

The Pre-Ph.D course work will be based on three papers

Paper-I

The scholars shall have to opt any one of the following courses in paper-I

a. **Recent advances in Theory of Polynomials**
b. **Recent advances in Graph Theory**
c. **Recent advances in Bio-Mathematical Modelling**
d. **Recent advances in Wavelet Theory**
e. **Recent advances in Variational Inequalities**

Paper-II (Research Methodology)

Paper-II shall be common to all the scholars of the Department.

Paper-III

Paper-III shall be based on the topic of the research.

Note:

There shall be four questions with internal choice in each paper carrying 25 marks each and the candidates shall be asked to do all the four questions within three hours.

Paper 1 (a): Recent Advances in Theory of Polynomials

Unit 1

Orthogonal polynomials, Classical Orthogonal polynomials, Recurrence relations, Symmetric functions (Polynomials) & Elementary symmetric functions Continuity theorem.

Unit II

Companion matrix, *Hardmard's* theorem, Greshgorean Disc theorem & its applications, Parodi's theorem & some other results, convex hull, Gauss Lucas theorem with application & its recent extensions due to Aziz & Zargar, Aziz & Rather, Polar Derivatives, analytic proof of Laguerre's theorem.

Unit III:

Apolar Polynomials, Grace's Theorem, Walsh's Coincidence Theorem, Recent Generalizations of Grace's Theorem due To Aziz, Szego's Composition Theorem, Cohn-Agervary Theorem and some related results due to A.Aziz .

Unit IV

Derivative estimates on the unit interval, Trigonometric polynomials, inequalities of *Bernstein* and *A. Markov*, Extension of *Markov's* inequality, Result of *Schur* and other extensions, *Bernstein's* theorem on the unit disk, some of its recent generalizations and refinements due to *Rahman* and *Schmeisser* and *A. Aziz.*

References:-

1. Theory of Functions, E.C.Titchmarsh.

2. Conformal Mappings, Z. Nehari.

3. Foundations of Complex Analysis, S.Ponnasamy .
4. Analytic Theory of Polynomials , Q.I.Rahman & Gerhard Schmeisser - 2002
   Oxford Science Publications.
5. Geometry of Polynomials, Morris Marden – Amer.Math.Soc, 1969.

Paper I(b): Recent Advances in Graph Theory

Maximum Marks: 100 Time 3 hours

Unit 1: Graph coloring, Vertex coloring, Brook's theorem, Edge coloring, Vizing's theorem, Region coloring, Tait's theorem, Matchings, Berge theorem, Hall's matching theorem, Tutte's 1-factor theorem, Edge(line) graphs, Characterization of edge graphs, Bieneke's theorem, Groups in graphs, Frucht's theorem, Cayley digraphs.

Unit 2: Perfect graphs, Perfect graph theorem, complete graphs are perfect, house graph is perfect, open chains are perfect, weak perfect graph theorem, strong perfect graph theorem SPGT (statement only), SPGT holds for circular arc graphs, comparability graph is perfect, Mirsky's theorem, incomparability graphs are perfect, Dilworth's theorem, Imperfect graphs, Chordal graphs, A graph G is chordaliff it has a perfect elimination ordering.

Unit 3: Flows and networks, Max-Flow-Min-Cut theorem,k- flow,nowhere-zero or positive flow, A graph has a nowhere-zero 2-flow iff it is an even graph, a cubic graph has a nowhere-zero 3-flow iff it is bipartite, example of Peterson graph, A plane bridgeless graph is k-face colorable iff it has nowhere-zero k-flow; If a cubic graph has a nowhere-zero 4-flow, then G is 3-edge colorable; A graph has a nowhere-zero 4-flow iff it is the union of two even graphs.

Unit 4: Spectra of graphs, Characteristic polynomial, Sach's theorem, spectrum of regular graphs, spectrum and walks, diameter, spanning trees, spectrum of bipartite graphs, interlacing, spectral radius of a graph. Energy of a graph, Coulson's integral formula, bounds for energy of a graph-Gutman; Koolen, energy of bipartite and regular graphs.

Radicals: Prime radical, primary radical, Nil radical, Jacobson radical, Nil radical is intersection of prime ideals. Noetherian rings, ascending chain conditions, Hilbert basis theorem. Artinian rings, descending chain conditions, Hopkin's theorem, homomorphic images of Noetherian and Artinian rings, radical of Artinian rings is the sum of all nilpotent ideals.

References:

1. M. F. Atiyah and I. G. MacDonald, Introduction to Commutative Algebra, Addison-Wesley.
2. R. B. Bapat, Graphs and Matrices, Springer.
3. G. Chartrand, Graphs and Digraphs, CRC Press.
4. P. M. Cohn, Introduction to Ring Theory, Springer.
5. D. Cvetkovic, M. Doob, H. Sachs, Spectra of Graphs, Theory and Applications, Academic Press.
6. C. Godsil, Gordon Royle, Algebraic Graph Theory, Springer.

7. I. Kaplanskey, Commutative Rings, The University of Chicago Press.
8. X. Li, Y. Shi, I. Gutman, Graph Energy, Springer.
9. S. Pirzada, An Introduction to Graph theory, Universities Press, OrientBlackSwan, 2012
10. D. B. West, Introduction to Graph Theory, Prentice Hall.

Paper-I (c): Recent advances in Bio-Mathematical Modelling

Max. Marks: 100

Unit I:
Mathematical modelling through differential equations, Linear and non-linear growth and decay models, Law of mass action, Compartment model, Diffusion of glucose or a medicine in the blood stream, Mathematical modelling through partial differential equations, Mass balance equation, Equation of continuity in fluid dynamics, Heat and mass flow models.

Unit II:
Momentum balance equation, PDE model for vibrating membrane, Mathematical modelling in terms of wave equation. Variational principles, Euler-Lagrange equation. The nature of PDE for the potential of the steady state two dimensional flow, Inviscid flow of ideal gas.

Unit III:
Energy transfer in human body, Energy conservation, Temperature response to human body, Modes of heat transfer, convective, conductive and radiative heat transfer, heat governing equations.Forced and natural convection.

Unit IV:
Mass conservation and kinetics, Gases and liquids, their equilibrium, rate laws of homogeneous equations. Zeroth, first and nth order reactions.Steady and unsteady state mass diffusions.

References:

1. Mathematical Modelling; JN Kapur; New Age International Publishers, New Delhi .
2. Introduction to Mathematical Modelling and Biomathematics; M. A. Khanday; Dilpreet Publishing House , New Delhi .
3. Mathematical Models of Biological Systems; Hugo van den Berg; Oxford University Press.
4. Conduction of Heat in Solids; H S Carslaw, J C Jarger; Oxford University Press.
5. The Mathematics Of Diffusion; J Crank; Oxford.
6. ATextbook on Heat Transfer; S P Sukhatme.

Paper-I (d): Recent advances in Wavelet Theory

Max. Marks: 100

Unit-I Wavelet Transforms

Motivation and definition of continuous wavelet transforms (CWT), basic properties of wavelet transforms, Haar wavelet, Mexican hat wavelet, Meyer wavelet and their Fourier transforms, Parseval's formula and energy preserving relation of CWT, reconstruction formula for CWT, fractional wavelet transform (FrWT), basic properties FrWT, examples of fractional wavelets (Mexican hat wavelet, Morlet wavelet).

Unit-II Construction and Characterization of MRA Wavelets

Motivation, definition and examples of multiresolution analysis (MRA) with special reference to Haar MRA and Shannon's MRA, properties and characterizations of scaling functions, construction of orthonormal wavelet bases in $L^2(R)$, characterization of orthonormal wavelets via Fourier transforms, dimension function.

Unit-III Other Constructions of Wavelets

Introduction to Splines, construction of *B*-spline wavelets, Franklin and Battle-Lemarie wavelets, construction of compactly supported wavelets (Daubechie's wavelets), biorthogonal multiresolution analysis, construction of biorthogonal wavelets, fractional multiresolution analysis, fractional wavelets.

Unit-IV Construction of Wavelet Frames

Definition and examples of wavelet frames, necessary and sufficient condition for wavelet frames in $L^2(R)$, tight wavelet frames, construction of MRA wavelet frames, unitary extension principle for wavelet frames.

Recommended Books:

1. L. Debnath and Firdous A. Shah, *Lecture Notes on Wavelet Transforms,* Birkhauser, Boston, 2017.
2. L. Debnath and Firdous A. Shah, *Wavelet Transforms and Their Applications,* Birkhauser, New York, 2015.
3. E. Hernandez and G. Weiss, *A First Course on Wavelets*, CRC Press, New York (1996).
4. D. K. Ruch and P. J. Van Fleet, *Wavelet Theory,* John Wiley, 2009.
5. P. Nickolas, Wavelets: A Students Guide, Cambridge University Press, 2017.
6. M. Pinsky, *Introduction to Fourier Analysis and Wavelets,* Brooks/Cole Publishing, 2002.
7. D. F. Walnut, *An introduction to Wavelet Analysis,* Birkhauser, New York, 2002.

8. O. Christensen, *An Introduction to Frames and Reisz Bases*, Birkhauser, New York, 2016.

Research Papers:

1. A. Prasad, S. Manna, A. Mahato and V. K. Singh, The generalized continuous wavelet transform associated with the fractional Fourier transform. *J. Comput. Appl. Math., 259*, 660-671, 2014.

2. H. Dai, Z. Zheng and W. Wang, A new fractional wavelet transform, Commun. Nonlinear Sci. Numeric Simulations, 44, 19-36, 2017.

3. M. Bahri and Ryuichi, A variation on uncertainty principle and logarithmic uncertainty principle for continuous quaternion wavelet transforms, Abstract and Applied Analysis, Article ID 3795120, https://doi.org/10.1155/2017/3795120. 2017.

4. L. T. Rachdi and F. Meherzi, Continuous wavelet transform and uncertainty principle related to the spherical mean operator, Mediterr. J. Math. 14:11, 2017.DOI 10.1007/s00009-016-0834-11660-5446/17/010001-23.

5. F. A. Shah, Construction of wavelet packets on *p*-adic field, *Int. J. Wavelets Multiresolut.Inf. Process.* 7(5), 553–565, 2009.

6. F. A. Shah, Tight wavelet frames generated by the Walsh polynomials. *Int. J. Wavelets, Multiresolut. Inf. Process.* 11(6), 1350042 (15 pages), 2013.

7. F. A. Shah, Periodic wavelet frames on local fields of positive characteristic. *Numerical Functional Analysis and Optimization*, *37*(5), 603–627, 2016.

8. F. A. Shah and Abdullah, Nonuniform multiresolution analysis on local fields of positive characteristic, *Complex Analysis Operator Theory.* 9, 1589-1608, 2015.

9. F. A. Shah and Abdullah, Wave packet frames on local fields of positive characteristic, *Applied Mathematics and Computation*, *249*, 133–141, 2014.

10. F. A. Shah and L. Debnath, Dyadic wavelet frames on a half-line using the Walsh-Fourier transform, *Integ. Transf. Special Functions,* 22(7), 477–486, 2011.

11. Y. F. Shen, D. H. Yuan and S.Z. Yang, Generalized wavelet frames, *Sci. Sin. Math.,* 45, 349-363, 2015.

12. I. Daubechies, B. Han, A. Ron and Z. Shen, Framelets: MRA-based constructions of wavelet frames, *Appl. Comput. Harmon. Anal.* 14, 1–46, 2003.

13. L. Zang and W. Sun, Inequalties for wavelet frames, *Numer. Funct. Anal. Optimizat.* 31, 1090-1101, 2010.

<u>Paper-I (e)</u>: <u>(Recent Advances in Nonlinear Functional Analysis)</u>

Max. Marks: 100

Unit-I

Some Geometric Properties of Banach Spaces: Uniformly Convex Spaces, Strictly Convex Banach Spaces, Uniform Convexity, Strict Convexity, Smooth Spaces: The Modulus of Smoothness, Duality Between Spaces, Duality Maps in Banach Spaces: Duality Maps of Some Concrete Spaces, Basic Notions of Convex Analysis, p-uniformly Convex Spaces, Uniformly Convex Spaces

Unit-II

Banach's Fixed Point Theorem, Quasi-Nonexpansive Operators, ϕ-Contractions, Generalized ϕ-Contractions, The Krasnoselskij Iteration: Nonexpansive Operators in Hilbert Spaces, Strictly Pseudo-contractive Operators, Lipschitzian and Generalized Pseudo-contractive Operators, Pseudo ϕ-Contractive Operators,

Unit-III

The General Mann Iteration, Nonexpansive and Quasi-Nonexpansive Operators, The Ishikawa Iteration: Lipschitzian and Pseudo-Contractive Operators in Hilbert Spaces, Strongly Pseudo-Contractive Operators in Banach Spaces,Quasi-Nonexpansive Type Operators, The Equivalence Between Mann and Ishikawa Iterations, Mann and Ishikawa Iterations with Errors

Unit-IV

Bilinear Forms and its Applications, Lax-Milgram Lemma, Minimization of Functionals, Variational Inequalities, Relationship Between Abstract Minimization Problems and Variational Inequalities, Lions Stampacchia Theorem for Existence of Solution of Variational Inequality.

Suggested Readings

1. **C.Chidume:** *Geometric Properties of Banach Spaces and Nonlinear Iterations:Springer-Verlag, London, 2009.*
2. **V.Berinde:***Iterative Approximation of Fixed Points,*Springer-Verlag, Berlin Heidelberg 2007.
3. **A.H. Siddiqi, K. Ahmed and P. Manchanda:***Introduction to Functional Analysis with Applications*, Anamaya Publishers, New Delhi-2006.

Paper II: Research Methodology Max. Marks: 100

Unit I: Introduction and Motivation

i).Importance of research and different aspects related to it, Identification of a research problem, Plagiarism and its prevention, writing research grant proposals,Paper writing skills,Searching Google,Mathscinet,scopus,SCI , Impact factor,H-index,Google scholar.

ii).Different techniques of Proofs viz. mathematical induction, deduction, proof by contradiction,Inverse and converse, contraposition, proof by construction, proof by counter examples.

Knowledge of Latex and Matlab.

Unit II: Matrices

Eigen values and eigen vectors of a matrix, Brouver's theorem,Companion matrix, Gereshgorian disc theorem, Positive semidefinite matrices,Fisher's inequality,Schur compliment and determinant inequalities, Hua determinant inequality, Courant-Fisher theorem,,Perturbation of eigen values of normal matrices, Hoffman-Weilandt theorem and Sun theorem.

Unit III: Numerical Methods

Finite Difference Calculus. Forward,Backward and Central differences, Differences of a Polynomial, Newton's and Lagrange's formulae for interpolation,Spline interpolations, Linear and Quadratic,Solution of heat,wave and Laplace equations by difference methods, Solution of ODE and PDE using finite differences.

Unit IV: Statistical Skills

Binomial, Poisson, normal, chi-square and t distributions, Correlation and regression theory, Statistical estimation theory, Statistical decision theory, Chi-square and t tests .

References

1. L.Lamport,LaTex,A Documentation System,2nd ed., Addison-Wesley,1994.

2. Frank Mittelbach, Michel Goosens,Johannes Braams, David Charlis, Chris Rowley, The LaTex

Companion,2nd ed.(TTCT series), Addison_Wesley,2004.

3. Nicholas J.Higham,Handbook of Writing for the mathematical Sciences,2nd ed.SIAM,1998.

4. Donald E.Knuth,Tracy L. larrabee,Paul M. Roberts,Mathematical Writing,Mathematical

Association of America Washington, D.C.,1989.

5. M.K.Jain, Computational techniques and Numerical methods.

6. S.S.Shastry, Introduction to Numerical Methods.

7. Franz E.Honz, Elementary matrix Algebra.

8. Rager A.Han and Johnson, matrix Analysis.

9. Mathematical Statistics, Hogg and Graig .

10. Mathematical Statistics.Gupta and Kapoor.

<u>Paper-III</u>: <u>Research Topics in Polynomial Theory (Adil and Abrar)</u>

Max. Marks: 100

Unit 1: <u>Inequalities for the Maximum Modulus of a Polynomial and its Derivative</u>

Bernstein's inequality, and some recent developments in extremal properties of polynomials, maximum modulus of a polynomial with restricted zeros , Ankeny –Rivlin theorem and its various refinements and generalizations , T.J.Rivlin's theorem and its various generalizations .

Unit 2: <u>Bounds on the Uniform Norm of Polar Derivative of a Polynomial</u>

Some basic results of Aziz , Rahman and others on the polar derivative of a polynomial , inequalities for polar derivatives for restricted polynomial classes, rational functions and some inequalities for maximum modulus of rational function with prescribed poles .

Unit 3: <u>Extremal Problems for Restricted Polynomial Classes in $L^r - norm$</u>

$L^r - norm$ inequalities for polynomials , Zygmund's inequality, De-Bruijn's inequality , some recent generalizations and refinements of these theorems due to Aziz , Rather ,Mir , Dewan , Govil , Rahman and others.

Unit 4: <u>Bounds for the Moduli of Zeros of a Polynomial</u>.

Cauchy's Classical theorem for the zeros of polynomials and some of its recent generalizations, several refinements and extensions of Cauchy's theorem via matrix method, Enestrom-Kakeya theorem and its generalizations due to Joyal ,Labelle , Rahman , Govil and others.

References:

1): Approximation Theory in memory of A.K.Varma, Monographs and Textbook in Pure and Applied Mathematics, CRC Press , Taylor and Francis Group, Boca Raton , London, New York, 2015

2): Analytic Theory of Polynomials, Q,.I . Rahman and G,Schmeisser, Oxford University Press, 2002.

3): Morris .Marden, Geometry of Polynomials, 2nd ed. AMS,Providence R.I, 1966.

4): A. Aziz, inequalities for the polar derivative of a polynomial, J.Approx. Theory 55 (1988), 183-193

5): A.Aziz and N.A Rather, refinement of theorem of Paul Turan concerning polynomials, Math. Ineq. Appl. 1 (1998), 231 - 238

6): A. Mir and S.A. Baba, Some inequalities for the polar derivative of a polynomial, Anal. Theory Appl. 27 (2011), 231-237.

Paper III: Research Topics in Wavelet Analysis **(Azhar, Irfan)**

Time Allowed: 03 hrs **Maximum Marks: 100**

Unit-I Wavelet Transforms

Motivation and definition of continuous wavelet transforms (CWT), basic properties of wavelet transforms, Haar wavelet, Mexican hat wavelet, Meyer wavelet and their Fourier transforms, Parseval's formula and energy preserving relation of CWT, reconstruction formula for CWT, fractional wavelet transform (FrWT), basic properties FrWT, examples of fractional wavelets (Mexican hat wavelet, Morlet wavelet).

Unit-II Construction and Characterization of MRA Wavelets

Motivation, definition and examples of multiresolution analysis (MRA) with special reference to Haar MRA and Shannon's MRA, properties and characterizations of scaling functions, construction of orthonormal wavelet bases in $L^2(R)$, characterization of orthonormal wavelets via Fourier transforms, dimension function.

Unit-III Other Constructions of Wavelets

Introduction to Splines, construction of *B*-spline wavelets, Franklin and Battle-Lemarie wavelets, construction of compactly supported wavelets (Daubechie's wavelets), biorthogonal multiresolution analysis, construction of biorthogonal wavelets, fractional multiresolution analysis, fractional wavelets.

Unit-IV Construction of Wavelet Frames

Definition and examples of wavelet frames, necessary and sufficient condition for wavelet frames in $L^2(R)$, tight wavelet frames, construction of MRA wavelet frames, unitary extension principle for wavelet frames.

Recommended Books:

1. L. Debnath and Firdous A. Shah, *Lecture Notes on Wavelet Transforms,* Birkhauser, Boston, 2017.
2. L. Debnath and Firdous A. Shah, *Wavelet Transforms and Their Applications,* Birkhauser, New York, 2015.
3. E. Hernandez and G. Weiss, *A First Course on Wavelets*, CRC Press, New York (1996).
4. D. K. Ruch and P. J. Van Fleet, *Wavelet Theory,* John Wiley, 2009.
5. P. Nickolas, Wavelets: A Students Guide, Cambridge University Press, 2017.
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7. D. F. Walnut, *An introduction to Wavelet Analysis,* Birkhauser, New York, 2002.
8. O. Christensen, *An Introduction to Frames and Reisz Bases*, Birkhauser, New York, 2016.

Research Papers:

1. A. Prasad, S. Manna, A. Mahato and V. K. Singh, The generalized continuous wavelet transform associated with the fractional Fourier transform. *J. Comput. Appl. Math., 259*, 660-671, 2014.
2. H. Dai, Z. Zheng and W. Wang, A new fractional wavelet transform, Commun. Nonlinear Sci. Numeric Simulations, 44, 19-36, 2017.
3. M. Bahri and Ryuichi, A variation on uncertainty principle and logarithmic uncertainty principle for continuous quaternion wavelet transforms, Abstract and Applied Analysis, Article ID 3795120, https://doi.org/10.1155/2017/3795120. 2017.
4. L. T. Rachdi and F. Meherzi, Continuous wavelet transform and uncertainty principle related to the spherical mean operator, Mediterr. J. Math. 14:11, 2017.DOI 10.1007/s00009-016-0834-11660-5446/17/010001-23.
5. F. A. Shah, Construction of wavelet packets on *p*-adic field, *Int. J. Wavelets Multiresolut.Inf. Process.* 7(5), 553–565, 2009.
6. F. A. Shah, Tight wavelet frames generated by the Walsh polynomials. *Int. J. Wavelets, Multiresolut. Inf. Process.* 11(6), 1350042 (15 pages), 2013.
7. F. A. Shah, Periodic wavelet frames on local fields of positive characteristic. *Numerical Functional Analysis and Optimization*, *37*(5), 603–627, 2016.
8. F. A. Shah and Abdullah, Nonuniform multiresolution analysis on local fields of positive characteristic, *Complex Analysis Operator Theory.* 9, 1589-1608, 2015.
9. F. A. Shah and Abdullah, Wave packet frames on local fields of positive characteristic, *Applied Mathematics and Computation*, *249*, 133–141, 2014.
10. F. A. Shah and L. Debnath, Dyadic wavelet frames on a half-line using the Walsh-Fourier transform, *Integ. Transf. Special Functions,* 22(7), 477–486, 2011.
11. Y. F. Shen, D. H. Yuan and S.Z. Yang, Generalized wavelet frames, *Sci. Sin. Math.,* 45, 349-363, 2015.
12. I. Daubechies, B. Han, A. Ron and Z. Shen, Framelets: MRA-based constructions of wavelet frames, *Appl. Comput. Harmon. Anal.* 14, 1–46, 2003.
13. L. Zang and W. Sun, Inequalties for wavelet frames, *Numer. Funct. Anal. Optimizat.* 31, 1090-1101, 2010.

Paper-III (Research topics in Variational Inequalities) (Mudasir)

Paper-III: **Marks: 100**

Unit-I

Reisz-Representation Theorem, Projection Mappings and their Properties, Characterization of Projection onto Convex sets and their Geometrical Interpretation, Monotone Operators, Maximal Monotone Operator and their Properties, Fixed Points of Nonexpansive Operators, Common Fixed Points, Markov-Kakutani Common Fixed point Theorem, Monotone Sequences.

Unit-II

Different Types of Monotone Operators, Minty Theorem, Strong Notions of Monotonicity such as Para, Strict, Uniform and Strong monotonicity, Lipschitz Continuity, Mixed Lipschitz Continuity, Retraction, Nonexpansive Retraction, Sunny Retraction, Normalized Duality mapping, Uniformly Smooth and Uniformly Convex Banach Space, Modulus of Smoothness

Unit-III

Variational Inequalities in R^n, First Theorem about Variational Inequalities, Variational Inequality Problem, Some Problems which Lead to Variational Inequalities: Minimization Problem, One Dimensional Obstacle Problem, Variational Inequalities in Hilbert Spaces.

Unit-IV

Some Classes of Variational Inequalities, Existence of solution by Projection Method, Wiener-Hopf Equation Method, Some classes of Variational Inclusions, Solution by Proximal Mapping, P-η-Proximal Mapping, Iterative Approximation of some classes of variational inequalities/inclusions considered by some authors, Stability of Iteration Procedures, Some set-valued generalizations.

Suggested Readings

1. **H. H. Bauschke and P.L. Combettes,***Convex Analysis and Monotone Operator Theory in Hilbert Spaces*, Springer New York, 2011.
2. **J.P. Aubin and A. Frankowska:***Set-Valued Analysis*, Birkh¨auser, Berlin, 1990.
3. **D. Kinderlehrer and G. Stampacchia:***An Introduction to Variational Inequalities and Their Applications*, Academic Press, New York, 1980.
4. **Cioranescu. I.,** Geometry of Banach Spaces, Duality Mappings and Nonlinear Problems, Kluwer Academic Publishers, Dordrecht, 1990
5. **M.C. Joshi and R.K. Bose:***Nonlinear Functional Analysis and its Applications,* Willey Eastern Limited.
6. **A.H. Siddiqi:***Recent Developments in Applicable Mathematics*, Macmillan India Limited, 1994.

Research Papers

1. **J. Deepho, P. Thounthong, P. Kumam, S. Phiangsungnoeng**, A new general iterative scheme for split variational inclusion and fixed point problems of k-strict pseudo-contraction mappings with convergence analysis, Journal of Computational and Applied Mathematics 318 (2017) 293–306.
2. **Y. Shehu, Y, Mewomo , O.T. , Ogbuisi, F.U. ,** Further investigation into approximation of a common solution of fixed point problems and split feasibility problems,*ActaMathematicaScientia 2016,36B(3):913–93.*
3. **K. Sitthithakerngkiet, J. Deepho, P. Kumam,** A hybrid viscosity algorithm via modify the hybrid steepest descent method for solving the split variational inclusion in image reconstruction and fixed point problems, *Appl. Math.andComput, 250 (2015) 986–1001.*
4. **Hossein P.,** A general iterative method for finding common solutions of system of equilibrium problems, system of variational inequalities and fixed point problems, *Mathematical and Computer Modelling 55(2012), 1622–1638.*
5. **Kazmi, K.R, Bhat, M.I.,** Convergence and stability of iterative algorithms for generalized set-valued variational-like inclusions in Banach spaces, Appl. Math. Comput. 166 (2005), 164-180.
6. **Kazmi, K.R, Bhat, M.I, Ahmad, N.,** An iterative algorithm based on *M*-proximal mappings for a system of generalized implicit variational inclusions in Banach spaces, J. Comput. App. Math., 233 (2009), 361-371.
7. **Kazmi, K.R, Rizvi, H.S.,** A hybrid extragradient method for approximating the common solutions of a variational inequality, a system of variational inequalities , a mixed equilibrium problem and a fixed point problem, Appl. Math. Comput., 218(2012), 5439-5452.
8. **Kazmi, K.R, Khan, F.A, Shahzad, M.,** A system of generalized variational inclusions involving generalized H(.,.)-accretive mapping in real q-uniformly smooth Banach spaces, Appl. Math. Comput. 217 (2011), 9679- 9688.
9. **Kazmi, K.R, Rizvi, H.S.,**Krasnoselski-Mann type iterative method for hierarchial fixed point problem and split mixed equilibrium problem, Numer. Algor. DOI 10.1007/s11075017-0316 (2013).
10. **Lou, J., et al.,** Iterative methods for solving a system of variational inclusions, monotone operators in Banach spaces. Comput. Math. Appl., 55 (2008): 18321841.
11. **Luo, X.P. and Huang, N.J.,** (H,η)-monotone operators in Banach spaces with an application to variational inclusions. Appl. Math. Comput., 216 (2010), 1131-1139
12. **Saeidi, S.,** Iterative algorithms for finding common solutions of variational inequalities and systems of equilibrium problems and fixed points of families and semigroups of nonexpansive mappings, Nonlinear Anal. 70 (2009), 4195–4208.
13. **Moudafi, A.,** Split monotone variational inclusions, J. Optim. Theory Appl. 150(2011), 275–283.
14. **Sahu, D.R, Kang, S.M, Sagar,V.,** Iterative methods for hierarchical common fixed point problems and variational inequalities, Fixed Point Theory Appl., (2013), 2013:299.
15. **Sahu, D.R, Kang, S.M, Sagar, V.,** Approximation of common fixed points of a sequence of nearly nonexpansive mappings and solutions of variational inequality problems, Journal of Applied Mathematics, Volume 2012, Article ID 902437, 12 pages doi:10.1155/2012/902437.

Paper III: Research topics on spectral graph theory (Hilal, Rizwan)

Maximum Marks: 100 Time 3 hours

Unit 1

Eigen values of the Laplacian of a graph (Anderson-Morley), on the Laplacian eigenvalues of a graph(Li-Zhang), a note on Laplacian eigenvalues of a graph (Merris), The second largest Laplacian eigenvalues of graphs, On Laplacian eigenvalues of a graph (Zhao), a characterization on graphs which achieve upper bound (Das). Laplacian spectrum of a graph (Das), a sharp upper bound on the largest eigenvalue of Laplacian matrix of graph (Shuet.al), Laplacian energy (Gutman-Zhao), bounds for Laplacian energy (Gutman-Zhao, Das-Mojallal)

- W.N. Anderson, T.D. Morley, Eigen values of the Laplacian of a graph, Linear and Multilinear algebra 18(1985) 141-145.
- J.S. Li, X.D. Zhang, On the Laplacian eigenvalues of a graph, Linear Algebra Applications, 285 (1998) 305-307.
- R. Merris, A note on Laplacian graph eigenvalues, Linear Algebra applications 285 (1998) 33-35.
- L.S. Li, Y.L. Pan, A note on the second largest eigenvalue of the Laplacian matrix of a graph, Linear Multilinear Algebra, 48 (2000) 117-121.
- I.M. Guo, On the second largest Laplacian eigenvalue of trees, Linear Algebra Applications 404 (2005) 251-261.
- J. L. Shu, Y. Hong, K.W. Ren, A sharp upper bound on the largest eigenvalue of Laplacian matrix of graph, Linear algebra application 347 (2002) 123-129
- B. Zhou,OnLaplacian eigenvalues of graph, Z. Naturforsch 59a(2004) 181-184.
- K.C. Das, A characterization on graphs which achieve upper bound for the largest Laplacian eigenvalue of graph,Linear algebra application, 376(2004)173-186.
- K.C. Das, the Laplacian spectrum of graph, computer and Mathematics with application, 48(2004)715-724.

Unit 2: On the sum of Laplacianeigen values of graph (Hammers et. al), on the sum of Laplacian eigenvalues of tree (Fritscher. et. al), on a conjecture for the sum of Laplacian eigenvalues (Wang et. al), an interlacing approach for bounding the sum (Abail et. al), on the distribution of Laplacian eigenvalues of tree (Braga et. al).

- W. H Heamers, A. Mohamadian, B. Tayfie-Rezaie, On the sum of Laplacian eigenvalues of graph, linear algebra application, 432(2010), 2214-2221.
- Ji.MingGuo, Xiao . Li. Wu, Jing. Ming. Zang, On the distribution of Laplacian eigenvalues of graph. Acta. Math. Sinica. English series. 27(11)(2011), 2259-2268.
- E. Fritscher, C. Hoppen, I. Rocha, V. Trevisan, On the sum of Laplacianeigen values of a tree, Linear Algebra Applications, 435(2011) 371-399.

- Z. Du, Bozhou, Upper bounds. For the sum of Laplacian eigenvalues of graphs, Lines Algebra Applications, 436 (2012) 3672-3683.
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- A. Abiad, M. A. Fiol, W.H. Haemers, G. Perarnau, An interlacing approach for bounding the sum of Laplacian eigenvalues of graphs.

- R.O.Braga, V.M.Rodrigues, V.Trevisan, On the distribution of Laplacian eigenvalues of trees, Discrete Math, 313 (2103) 2383-2389.

Unit 3: Bounding the sum of the largest Laplacian eigenvalues of graphs (Rocha et. al), on the Laplacian eigenvalues of a graph and Laplacian energy (Pirzada et. al), on the sum of the two largest Laplacian eigenvalues of unicycle graphs (Zheng et. al), Brouwer's conjecture (Pirzada et. al), distribution of Laplacian eigenvalues of graphs (Das et. al), Laplacian spread of a tree (Fan et.al)

- I. Rocha, V. Trevisan, Bounding the sum of largest LaplacianEigenvalues of graphs, Discrete Applied Math 170 (2014) 95-103.
- S. Pirzada, Hilal Ahmad Ganie, On the Laplacian Eigenvalues of a graph and Laplacian energy, Linear Algebra Applications 486 (2015) 454-468.
- Y. Zheng, An Chang, Jianxi Ki, On the sum of two largest eigenvalues of unicyclic graphs, J. Inequalities Applications (201&) 1-8.
- Hilab Ahmad Ganai, Ahmad M. Algamdi, S. Pirzada, On the sum of Laplacian Eigenvalues of a graph and Brouwer's conjecture, Linear Algebra Applications, 510(2016) 376-389.
- K.C. Das, S.A.Mojallal, V. Trevisan, Distribution of Laplacian eigenvalues of graphs. Linear Algebra Applications 508 (2016) 48-61.
- Yi- Zheng Fan, J. Xu, Y. Wang, D. Liang, the Laplacian spread of a tree, Discrete Math theoretical computer science 10(1)(2008) 79-86.

Unit 4: Recent developments on Laplacian energy.

Recent bounds for Laplacian energy (Das-Mojallal-Gutman; Pirzada- Ganai; Ganai-Algamdi-Pirzada), unicyclic graphs with equal Laplacianenergy(Fritscher-Hoppen-Trevison), Laplacian energy of threshold graphs (Vinagre-Vecchio-Justi-Trevison; Das-Mojallal; Helmberg-Trevison)

References:

1. R. B. Bapat, Graphs and Matrices, Springer.
2. G. Chartrand, Graphs and Digraphs, CRC Press.
3. D. Cvetkovic, M. Doob, H. Sachs, Spectra of Graphs, Theory and Applications, Academic Press.
4. C. Godsil, Gordon Royle, Algebraic Graph Theory, Springer.
5. X. Li, Y. Shi, I. Gutman, Graph Energy, Springer.
6. S. Pirzada, An Introduction to Graph theory, Universities Press, OrientBlackSwan, 2012 7. D. B. West, Introduction to Graph Theory, Prentice Hall.

Paper-III : Research Topics in Mathematical Physiology (Saqib, Ahsan)

Max. Marks: 100

Unit I:

System physiology; the circulatory system, blood flow, cardiac output; Simple and linear compartment circulatory system, Stewart- Hamilton method for measuring cardic output, the conservation laws, the Windkessel model, a small amplitude pulse wave, shock waves in the Aorta.

Unit II:

Blood plasma, blood cell production, periodic hematoligical disease, a simple model for blood cell growth, erythrocytes, myoglobin and haemoglobin, saturation shifts, CO_2 transport, Leukocytes, Leukocyte chemotaxis.

Unit III:

Respiration, Capillary-Alveoli gas exchange, diffusion across an interface, Capillary-Alveoli transport: CO_2 Removal and Oxygen uptake, CO Poisoning, ventilation and perfusion, the oxygen-carbondioxide diagram, respiratory exchange ratio, detailed model of respiratory regulation; Hypoxia and other issues.

Unit IV:

Law of mass action, thermodynamics and rate constants, enzyme kinetics, equilibrium approximation and quasi-steady state approximation, enzyme inhibition and cooperativity, reversible enzyme reactions.

Cellular Homeostasis, the cell membrane, Diffusion: Fick`s laws, diffusion through a membrane, diffusion into a capillary, Transdermal drug diffusion.

References:

14. Mathematical Models of Biological Systems; Hugo van den Berg; Oxford University Press.
15. Introduction to Mathematical Modelling and Biomathematics; M. A. Khanday; Dilpreet Publishing House , New Delhi.
16. Mathematical Models in Biology; Elizabeth S Allman, John A Rhodes.
17. A Textbook of Medical Physiology 8th edition; Guyton; W B Saunders Pub.
18. Emerging Topics In Heat and Mass Transfer in Porous Media; Peter Vadasz; Springer.
19. Essiantials of Human Physiology ; Noel Paton ; W T Keener and Co, Chicago.
20. Biological and Bioenvironmental Heat and Mass Transfer; Ashim K Datta; New York, Basel.
21. Mathematical Physiology Vol. I-III ; James Keener, James Sneyd; Springer.

(Aaqib Iqbal)

Paper 3rd : Research Topics in polynomial theory

Unit I: Bounds for the moduli of the zeros of polynomials. Recent generalizations of *Cauchy's* theorem and related results. Some recent generalizations of *Eneström-Kakeya* theorem. Results of *A. Aziz*, *A. Aziz & Mohmmad*, *Dewan et.al*, *A. Aziz* and *N.A. Rather*, *Aziz* and *Zargar* and others.

UNIT II: Some recent results in extremal properties of polynomials, growth of polynomials and inequalities for derivatives of the polynomial. Results of *Bernstein*, *Erdos-Lax, Malik*, *Rivlin*, *Govil*, *Aziz* and others.

UNIT III: L_p – Norm inequalities for polynomials, *Zygmund's* inequality, *De-Bruijin's* theorem, recent generalizations and refinements of these theorems due to *A. Aziz, A. Aziz* and *N.A. Rather, Govil, Govil* and *Rahman, Rahman* and *G. Schmeisser*.

UNIT IV: Operator preserving inequalities between polynomials, B_n-operator, inequalities for maximum modulus of the B_n-operator. L_p-norm inequalities for B_n-operators.

REFERENCES

- A. Aziz, inequalities for the polar derivatives of a polynomial, J. Approx. Theory 55 (1988), 183-193.
- A. Aziz and N. A. Rather, Location of the zeros of the trinomials and quadrinomials, Mathematical Inequalities & Applications, 17(3),(2014), 823-829.
- A. Aziz and N. A. Rather, Bounds for the zeros of class of lacunary-type polynomials, Mathematical Inequalities & Applications, Vol., 7(2013), 445 – 452.
- A. Aziz and N.A Rather, Some new generalizations of Zygmund type inequalities for polynomials, Mathematical Inequalities & Applications, 15(2012),469-486.
- A. Aziz and N.A Rather, Some Zygmund type L^q inequalities for polynomials, J. Math. Anal & Appl., 289(2004), 14-29
- A. Aziz and Q. Aliya, Estimates for the moduli of the zeros of a polynomial, Mathematical Inequalities & Applications, Vol. 9, No. 1, (2006), 107-116.
- A. Aziz and Q.G. Mohammad, On the zeros of certain class of polynomials and related analytic functions, J. Math. Anal. Appl., 75(1980), 495-502.
- A. Aziz, N.A.Rather and Q. Aliya, L_q Norm Inequalities for the polar derivatives of a polynomial, Mathematical Inequalities & Applications, Vol. 11, Number 2 (2008), 283-296.
- K.K. Dewan, Inequalities for a polynomial and its derivative *II*, J.Math. Anal. Appl., 190(1995), 625-629.

- A. Aziz and Q.G. Mohammad, Zero free regions for polynomials and some generalizations of Eneström-Kakeya theorem, Canad. Math. Bull.,27(1984), 265-272.
- R.B.Gardner and A. Weems, Bernstein type L_p Inequality for a certain class of polynomials, J.Math. Anal. Appl., 219(1998), 472-478.
- M.A.Qazi, On the maximum modulus of polynomials, Proc. Amer. Math. Soc. 115(1992), 337-343.
- Q.I.Rahman and G.Schmeisser, L_p inequalities for polynomials, J. Approx. Theory, 53(1988), 26-32.
- N.A Rather Some integral inequalities for polynomials, The South East Asian Bulletin of Mathematics, 33(2009), 341-348.
- N.A. Rather and M.A. Shah, on an operator preserving inequalities between polynomials, J. Math. Anal. & Appl.,399(2013), 422-432.

(Ishfaq Dar)

Paper 3rd : Research topics in polynomial theory

Unit I: Extremal properties of polynomials, Growth of polynomials and inequalities for derivatives of the polynomial. Results of Bernstein, Erdos-Lax, Malik, Rivlin, Govil, A. Aziz and others.

UNIT II: L_p – Norm inequalities for polynomials, Zygmund's inequality, De-Bruijin's theorem, recent generalizations and refinements of these theorems due to A. Aziz, A. Aziz and N.A. Rather, Govil, Govil and Rahman, Rahman and G. Schmeisser.

UNIT III: Polar derivatives of a polynomial, Laguerre's theorem and its applications. Inequalities for polar derivatives of a polynomial with restricted zeros due to A. Aziz and others.

UNIT IV: Cauchy's theorem and its recent generalizations and related results. Some recent generalizations of Eneströme-Kakeya theorem. Results of A. Aziz, A. Aziz and Mohmmad, Dewan et.al, A. Aziz and N.A. Rather, A. Aziz & Zargar, Govil and others.

REFERENCES

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- A. Aziz and N.A Rather, Some new generalizations of Zygmund type inequalities for polynomials, Mathematical Inequalities & Applications, 15(2012), 469-486.
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